PROBLEM 2 Saving Up

Marcus and Phillip are in the Robotics Club. They are both saving money to buy materials to build a new robot. They plan to save the same amount of money each week.

1. Write a function to represent the time it takes Marcus and Phillip to save money. Define your variables and explain why you chose those variables.

   w = time (in weeks)

   $M(w)$ = Marcus's savings (in dollars)

   $P(w)$ = Phillip's savings (in dollars)

   $M(w) = P(w)$

Marcus decides to open a new bank account. He deposits \$25 that he won in a robotics competition. He also plans on depositing \$10 a week that he earns from tutoring. Phillip decides he wants to keep his money in a sock drawer. He already has \$40 saved from mowing lawns over the summer. He plans to also save \$10 a week from his allowance.

2. Write a function to represent the information regarding Marcus and Phillip saving money for new robotics materials.

   $M(w) = 25 + 10w$

   $P(w) = 40 + 10w$

3. Predict when Marcus and Phillip will have the same amount of money saved. Use your functions to help you determine your prediction.

   Never. They are saving the same amount each week, but Phillip starts with \$40 and Marcus starts with \$25.

You can prove your prediction by solving and graphing a system of linear equations.

4. Rewrite each function as an equation. Use *x* and *y* for the variables of each function in equation form and define the variables. Then, write a system of linear equations.

   x = time (in weeks)

   y = savings (in dollars)

   $y = 25 + 10x$

   $y = 40 + 10x$

5. Analyze each equation.

a. Describe what the slope of each line represents in this problem situation.

   The amount of money each person saves each week.

b. How do the slopes compare? Describe what this means in terms of this problem situation.

   The slopes are the same. They each save $10/week.

c. Describe what the *y*-intercept of each line represents in this problem situation.

   The amount of money each person had in the beginning.

d. How do the *y*-intercepts compare? Describe what this means in terms of this problem situation.

   Phillip's y-intercept > Marcus's y-intercept because Phillip started with $40 and Marcus started with $25.

6. Determine the solution of the system of linear equations algebraically and graphically.

a. Use the substitution method to determine the intersection point.

To solve algebraically…

$y = 25 + 10x$
$y = 40 + 10x$

Set the two equations equal to each other…

$$\begin{array}{r} 25 + 10x = 40 + 10x \\ -10x \qquad\quad -10x \\ \hline 25 \neq 40 \end{array}$$

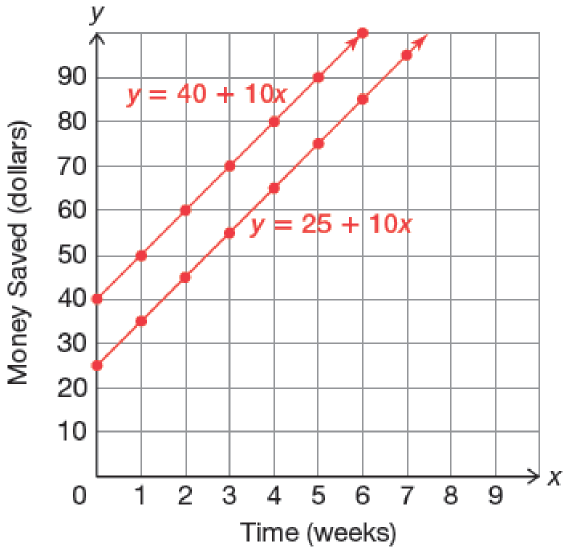
b. Does your solution make sense? Describe what this means in terms of the problem situation.

No, $25 \neq 40$ so there is NO SOLUTION.

c. Predict what the graph of this system will look like. Explain your reasoning.

If there is no solution, the lines must be PARALLEL, meaning same slope, but different *y*-intercepts.

d. Graph both equations on the coordinate plane provided.



7. Analyze the graph you created.

a. Describe the relationship between the graphs.

The lines are parallel.

b. Does this linear system have a solution? Explain your reasoning.

No. The graphs never intersect so there is no solution.

8. Was your prediction in Question 3 correct? Explain how you algebraically and graphically proved your prediction.

Yes. Algebraically, $25 \neq 40$ so there is no solution.

Graphically, the lines never intersect so there is no solution.

Skip Problems 9 & 10. Go to Page 375.

11. Phillip and Tonya went on a shopping spree this weekend and spent all their savings except for $40 each. Phillip is still saving $10 a week from his allowance. Tonya now deposits her tips twice a week. On Tuesdays she deposits $4 and on Saturdays she deposits $6. Phillip claims he is still saving more each week than Tonya.

a. Do you think Phillip's claim is true? Explain your reasoning.

No. Phillip and Tonya save the same amount of money each week. $4 + $6 = $10.
They also have the same amount of money left in savings. $40

b. How can you prove your prediction?

By writing a system of linear equations and solving them algebraically and graphically.

12. Prove your prediction algebraically and graphically.

a. Write ***functions*** that represent any new information about the way Tonya and Phillip are now saving money.

$$T(w) = 40 + 6w + 4w$$

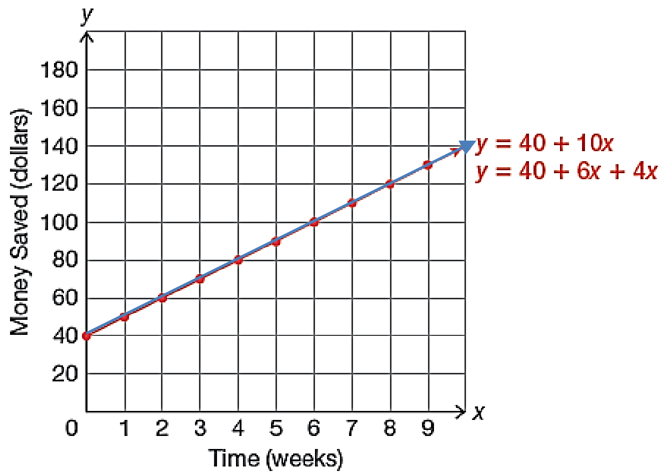
$$P(w) = 40 + 10w$$

b. Write a new ***linear system*** to represent the total amount of money each friend has after a certain amount of time.

$$y = 40 + 6x + 4x$$

$$y = 40 + 10x$$

c. Graph the linear system on the coordinate plane.



13. Analyze the graph.

a. Describe the relationship between the graphs. What does this mean in terms of this problem situation?

They are the same graph. Tonya and Phillip will always have the same amount of money.

b. Algebraically prove the relationship you stated in part (a).

$$40+10x=40+6x+4x$$
$$40+10x=40+10x$$
$$40=40$$

c. Does this solution prove the relationship? Explain your reasoning.

Yes, 40 = 40 means there are INFINITE SOLUTIONS.

Therefore, the graphs must be the same. Same slope and same *y*-intercept.

14. Was Phillip's claim that he is still saving more than Tonya a true statement? Explain why or why not.

No. Phillip is saving the same amount as Tonya.