6.1

Prepping for the Robot Challenge

Solving Linear Systems Graphically and Algebraically

LEARNING GOALS

In this lesson, you will:

- Write systems of linear equations.
- Graph systems of linear equations.
- Determine the intersection point, or break-even point, from a graph.
- Use the substitution method to determine the intersection point.
- Understand that systems of equations can have one, zero, or infinite solutions.

KEY TERMS

- break-even point
- system of linear equations
- substitution method
- consistent systems
- inconsistent systems

PROBLEM 1 Gearing For Success

Gwen has a part-time job working at Reliable Robots (RR) which sells electronics and hardware parts for robot creators. One of her tasks is to analyze RR's finances in terms of cost and income. Her boss, Mr. Robo, asks her to determine the *break-even point* for the cost and the income. The **break-even point** is the point when the cost and the income are equal. Gwen begins with the income and costs for gearboxes.

1. Let the function $I(g)$ represent the income (I) from selling gearboxes (g) and the function $C(g)$ represent the cost (C) of purchasing gearboxes (g).

   a. Describe the relationship between the income function and the cost function that will show the break-even point. Explain your reasoning.

   $I(g) = C(g)$

   The break-even point is when COST = INCOME.

   b. Describe the relationship between the income function and the cost function that will show a profit from selling gearboxes. Explain your reasoning.

   $I(g) > C(g)$

   Profit is when INCOME > COST.

2. RR purchases gearboxes from The Metalists for \$5.77 per gearbox plus a one-time credit check fee of \$45.00. RR sells each gearbox for \$8.50.

g = # of gear boxes

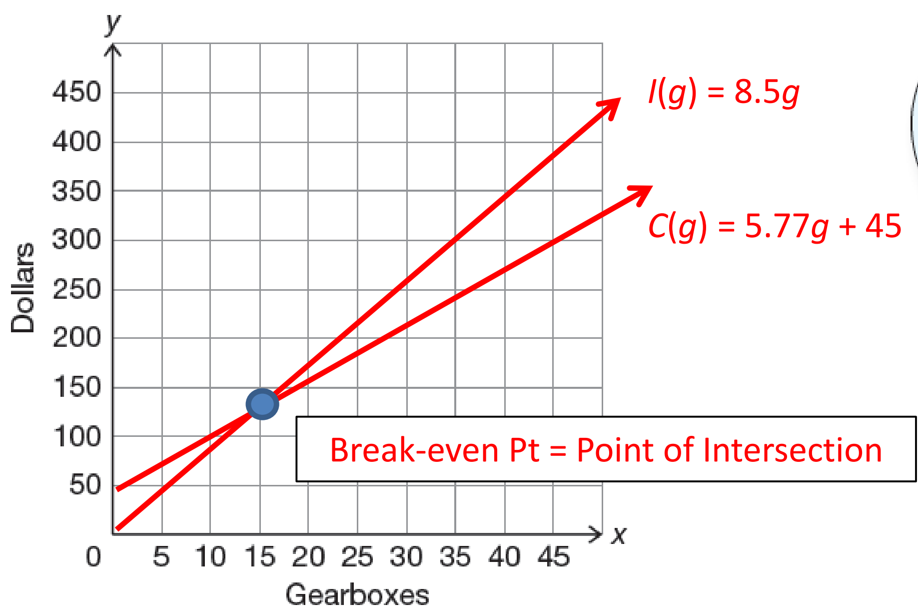
a. Write the function for the income generated from selling gearboxes.

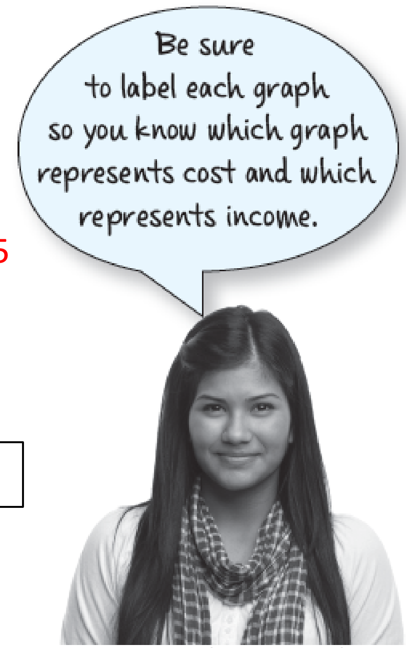
$I(g) = 8.5g$

b. Write the function for the cost of purchasing gearboxes from The Metalists.

$C(g) = 5.77g + 45$

3. Sketch a graph of each function on the coordinate plane to predict the break-even point of the income from RR selling the gearboxes and the cost of purchasing the gearboxes.





a. How is the break-even point for $I(g)$ and $C(g)$ represented on the graph you sketched? Estimate the break-even point.

It is where the 2 lines intersect near the point (15, 125) or between 15 and 20 gearboxes. So, RR must sell about 15 gearboxes to break-even.

b. Could you determine the exact break-even point from the graph? Why or why not.

No, the 2 lines do not intersect at an exact location on the *x*- and *y*-axes.

As you learned previously, the coordinates of an intersection of two graphs can be exact or approximate depending on whether the intersection point is located on the intersection of two grid lines. You also learned that you had to use algebra to prove an exact intersection point.

When determining the break-even point algebraically between two functions, it is more efficient to transform each function into equation form. In this case, by transforming the functions into equation form, you establish one unit of measure for the dependent quantity: dollars.

Analyze the functions representing cost and income from gearboxes for Reliable Robots.

$$I(g) = 8.5g \qquad C(g) = 5.77g + 45$$

Since g is the independent variable, you can represent g as x in equation form.

$$I(x) = 8.5x \qquad C(x) = 5.77x + 45$$

Since both $I(g)$ and $C(g)$ represent the dependent quantity in dollars, you can represent each using y as the variable.

$$y = 8.5x \qquad y = 5.77x + 45$$

4. Do you think it is possible to use other variables instead of *x* and *y* when transforming a function written in function notation to equation form?

Yes. You can use any variables you want as long as you choose the same variables to represent the independent and dependent quantities in each equation.

When two or more equations define a relationship between quantities, they form a **system of linear equations**.

5. What is the relationship between the two equations in this problem situation?

One equation represents the INCOME earned by ***selling*** gearboxes.
The other equation represents the COST of ***buying*** the gearboxes.

Now that you have successfully created a system of linear equations, you can determine the break-even point for the gearboxes at RR. One way to solve a system of linear equations is called the ***substitution method***. The substitution method is a process of solving a system of equations by substituting a variable in one equation with an equivalent expression.

Consider the system of equations from the previous worked example.

***Important!!!

$$\begin{cases} y = 5.77x + 45 \\ y = 8.5x \end{cases}$$

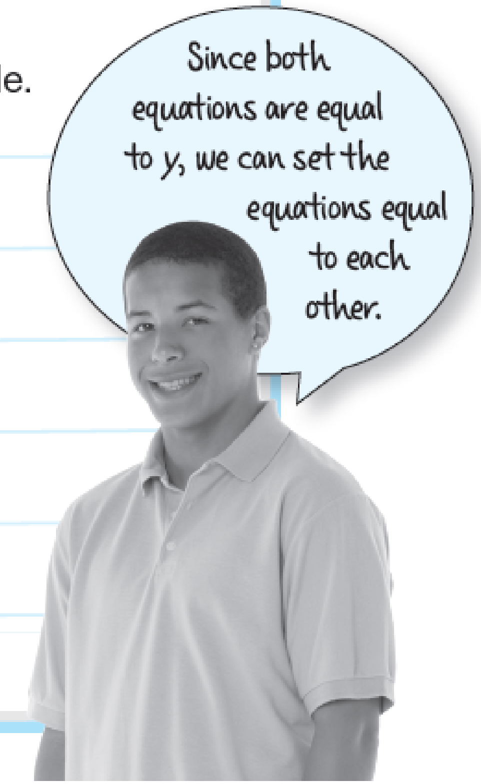
Substitute the variable y in the first equation with the equivalent expression in the second equation.

$$8.5x = 5.77x + 45$$

Isolate the variable to solve.

$$2.73x = 45$$

$$x \approx 16.48$$



6. Analyze the solution $x \approx 16.48$.

a. What does this point represent in terms of the problem situation? Why is this solution an approximation?

This point represents the # of gearboxes RR must sell before they make money.

The solution is ***rounded*** to the 100ths place.

b. Solve for y. Describe the solution in terms of this problem situation.

$y = 8.5x$

$y = 8.5(16.48)$

$y \approx 140.08$

Let x = 16.48 and solve for y.

The solution has two meanings. It represents the cost of buying the gearboxes and the income from selling the gearboxes.

c. What is the profit from gearboxes at the break-even point?

Profit = $0

d. Does this break-even point make sense in terms of the problem situation? Why or why not.

No. RR cannot sell 16.48 gearboxes. They will have to sell 17 gearboxes.