



**Learning Goal:**

To solve a system of equations algebraically using linear combinations (elimination).

Solving Systems of Equations

- 1) Use **graphing** to get an approximate answer or if the lines are easy to graph, i.e. slope-intercept form.
- 2) Use **substitution** if one variable can be easily replaced by its value or an expression that includes the other variable, i.e.  $y =$  or  $x =$ .
- 3) Use **linear combinations** when it is easy to eliminate a variable by **adding** or **subtracting** the system of equations.

Solving a System of Equations Using Linear Combinations

1. **Stack the system of equations** so common terms (like  $x$  and  $y$ ) line up.
2. **Choose which variable to eliminate.** The coefficients should be equal, but with opposite signs.
  - a. Does one of the variables have the same coefficient in both equations?
  - b. Can you multiply one or both equations by a number so one of the variables will have the same coefficient in both equations? Hint: find the LCM (least common multiple).
3. **Add the system of equations** to eliminate one of the variables.
4. **Solve for one variable.**
5. Plug the solution into one of the equations to **solve for the other variable.**
6. **Write** your solution **as an ordered pair.**

Solving a System by Adding Equations

<b>Steps:</b>	<u><b>Example 1</b></u>
<ul style="list-style-type: none"><li>▪ Eliminate <math>y</math> by adding the system of equations.</li><li>▪ Solve for <math>x</math>.</li><li>▪ Replace the value of <math>x</math> in one of the equations to solve for <math>y</math>.</li></ul>	$\begin{array}{r} 2x + 5y = 17 \\ 6x - 5y = -9 \end{array}$ $\begin{array}{r} 2x + 5y = 17 \\ \underline{6x - 5y = -9} \\ 8x + 0 = 8 \\ 8x = 8 \\ x = 1 \end{array}$ <p>Since <math>5y + -5y = 0</math>, add the equations to eliminate <math>y</math>.</p> $\begin{array}{r} 2x + 5y = 17 \\ 2(1) + 5y = 17 \\ 2 + 5y = 17 \\ 5y = 15 \\ y = 3 \end{array}$ <p>The solution is <math>(1, 3)</math>.</p>

Let's Practice:

$$2x + 3y = 11$$

$$-2x + 9y = 1$$

The solution is (4, 1).

What if the 2<sup>nd</sup> equation was  $2x - 9y = -1$ ? How would you solve it?

Solving a System by Multiplying One Equation

Steps:

- Stack the equations so common terms line up.
- Multiply the 2<sup>nd</sup> equation by -3 so the coefficients of  $y$  are equal but with opposite signs.
- Eliminate  $y$  by adding the system of equations.
- Solve for  $x$ .
- Replace the value of  $x$  in one of the equations to solve for  $y$ .

Example 2

$$15y = 2x - 32$$
$$-7x + 5y = -17$$

$$\begin{array}{l} -2x + 15y = -32 \\ \underline{-7x + 5y = -17} \end{array} \rightarrow [-7x + 5y = -17] \times -3$$

$$\begin{array}{l} -2x + 15y = -32 \\ \underline{21x - 15y = 51} \\ 19x + 0 = 19 \\ 19x = 19 \\ x = 1 \end{array}$$

$$\begin{array}{l} -2x + 15y = -32 \\ -2(1) + 15y = -32 \\ -2 + 15y = -32 \\ 15y = -30 \\ y = -2 \end{array}$$

The solution is (1, -2).

Let's Practice:

$$6x + 3y = -6$$

$$-2x + 5y = 14$$

The solution is (-2, 2).

## Solving a System by Multiplying Both Equations

### Steps:

- Multiply the 1<sup>st</sup> equation by 3 and the 2<sup>nd</sup> equation by -2 so the coefficients of the same variable are the equal but with opposite signs.
- Eliminate  $y$  by adding the system of equations.
- Solve for  $x$ .
- Replace the value of  $x$  in one of the equations to solve for  $y$ .

### Example 3

$$3x + 2y = 1$$

$$4x + 3y = -2$$

$$\underline{3x + 2y = 1}$$

$$\underline{4x + 3y = -2}$$

$$\rightarrow [3x + 2y = 1] \times 3$$

$$\rightarrow [4x + 3y = -2] \times -2$$

$$9x + 6y = 3$$

$$\underline{-8x - 6y = 4}$$

$$x + 0 = 7$$

$$x = 7$$

$$3x + 2y = 1$$

$$3(7) + 2y = 1$$

$$21 + 2y = 1$$

$$2y = -20$$

$$y = -10$$

**The solution is (7, -10).**

### Let's Practice:

$$7x - 3y = -5$$

$$3x + 2y = 11$$

**The solution is (1, 4).**

*If you ELIMINATE both variables and you are left with a TRUE statement, then the system of equations has INFINITE SOLUTIONS. A FALSE statement means there is NO SOLUTION.*