

Write a system of linear equations to represent the "break-even point" for a business. Then, graph the system of equations. Use the graph to estimate the break-even point. Then, calculate the exact break-even point algebraically. Round to the 100th place. Explain what the break-even point represents with respect to each business.

1. Jayla sets up a lemonade stand in front of her house. Each cup of lemonade costs \$0.30 to make. She spends \$6 on the advertising signs she puts up around the neighborhood. She sells each cup of lemonade for \$1.50.

Income equation:

$$y = 1.5x$$

Expense equation:

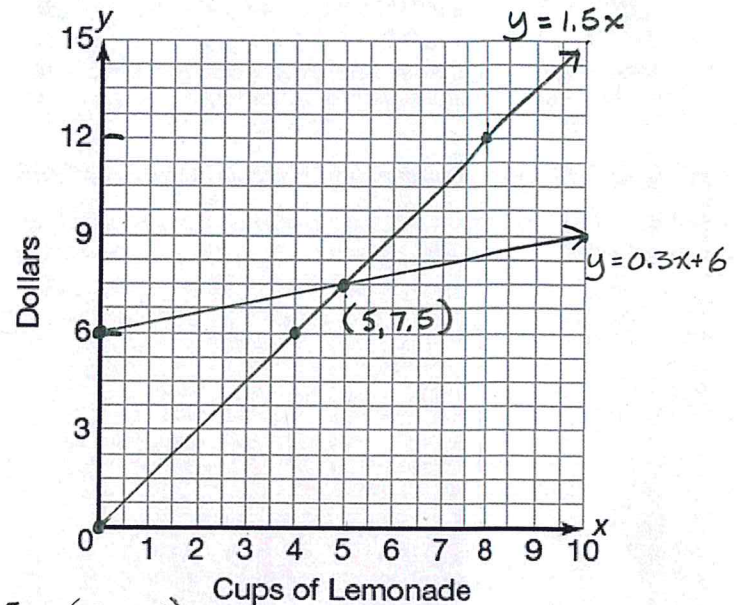
$$y = 0.3x + 6$$

Break-Even Point estimate (graphically):

$$(5, 7.5)$$

Break-Even Point exact (algebraically):

$$\begin{array}{r} y = y \\ 1.5x = 0.3x + 6 \\ - 0.3x \quad - 0.3x \\ \hline 1.2x = 6 \\ \frac{1.2x}{1.2} = \frac{6}{1.2} \\ x = 5 \\ y = 1.5(5) = 7.5 \quad (5, 7.5) \end{array}$$



What does the break-even point represent?

Jayla must sell 5 cups of lemonade to break-even. Her income and expenses are \$7.50.

2. Gabe starts his own lawn mowing business. He spends \$180 on a new lawnmower. For each yard he mows, he receives \$20, but spends \$4 on gas.

Income equation:

$$y = 20x$$

Expense equation:

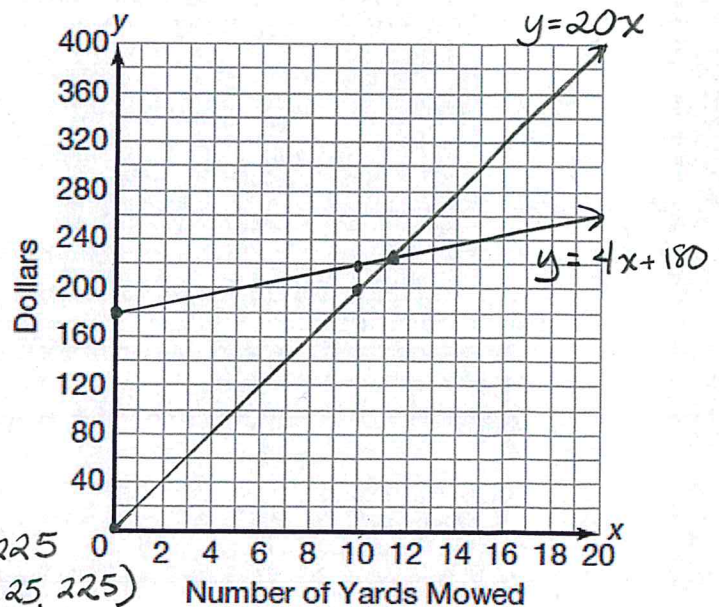
$$y = 4x + 180$$

Break-Even Point estimate:

x is between 11 and 12.
 y is between 220 and 240.

Break-Even Point exact:

$$\begin{array}{r} y = y \\ 20x = 4x + 180 \\ - 4x \quad - 4x \\ \hline 16x = 180 \\ \frac{16x}{16} = \frac{180}{16} \\ x = 11.25 \\ y = 20(11.25) = 225 \quad (11.25, 225) \end{array}$$



What does the break-even point represent?

Gabe must mow 12 yards to break-even. His income and expenses are \$225.

3. Merari is building birdhouses to raise money for a trip to Hawaii. She spends \$30 on the tools needed to build the houses. The material to build each birdhouse costs \$3.25. Merari sells each birdhouse for \$10.

Income equation:

$$y = 10x$$

Expense equation:

$$y = 3.25x + 30$$

Break-Even Point estimate:

x is between 4 and 5

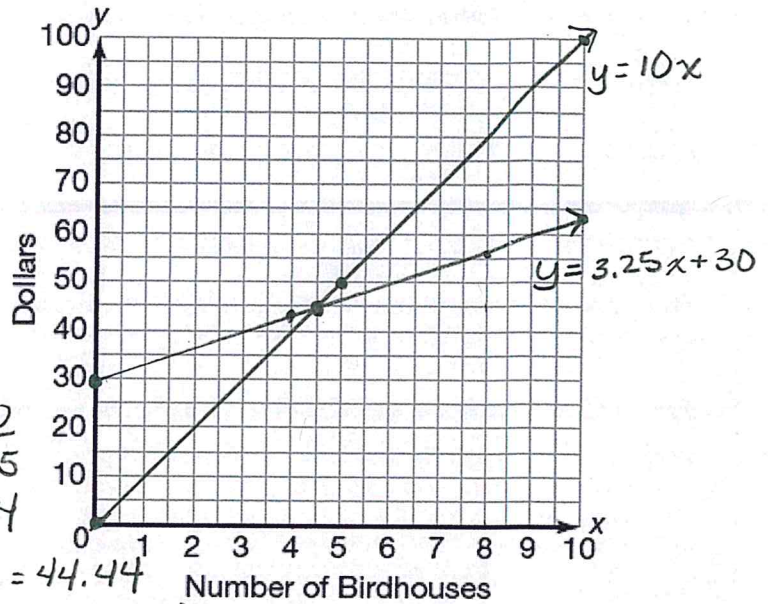
y is between 40 and 50.

Break-Even Point exact:

$$\begin{array}{r}
 y = y \\
 10x = 3.25x + 30 \\
 - 3.25x \quad - 3.25x \\
 \hline
 6.75x = 30 \\
 \frac{6.75x}{6.75} = \frac{30}{6.75} \\
 x = 4.44
 \end{array}$$

$$y = 10(4.44) = 44.44$$

$(4.44, 44.44)$



What does the break-even point represent?

Merari must build 5 birdhouses to break-even. Her income and expenses are \$44.44

4. The Spanish Club is selling boxes of fruit as a fundraiser. The fruit company charges the Spanish Club \$7.50 for each box of fruit and a shipping and handling fee of \$100 for the entire order. The Spanish Club sells each box of fruit for \$15.

Income equation:

$$y = 15x$$

Expense equation:

$$y = 7.5x + 100$$

Break-Even Point estimate:

x is between 12 and 14

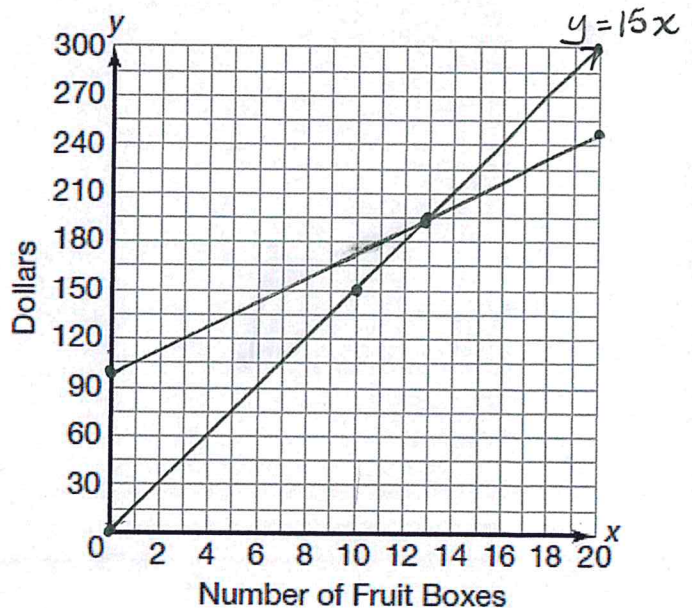
y is between 180 and 210.

Break-Even Point exact:

$$\begin{array}{r}
 y = y \\
 15x = 7.5x + 100 \\
 - 7.5x \quad - 7.5x \\
 \hline
 7.5x = 100 \\
 \frac{7.5x}{7.5} = \frac{100}{7.5} \\
 x = 13.33
 \end{array}$$

$$y = 15(13.33)$$

$y = 200 \quad (13.33, 200)$



What does the break-even point represent?

The Spanish Club must sell 14 boxes of fruit to break-even. Their income and expenses are \$200.