5.3 Let the Transformations Begin!

Translations of Linear and Exponential Functions

LEARNING GOALS

In this lesson, you will:

- Translate linear and exponential functions vertically.
- Translate linear and exponential functions horizontally.

KEY TERMS

- basic function
- transformation
- vertical translation
- coordinate notation
- argument of a function
- horizontal translation

PROBLEM 1 Vertical Translations

Consider the three linear functions shown.

- $g(x) = x$
- $c(x) = (x) + 3$
- $d(x) = (x) - 3$

Also, known as the "parent function"

The first function is the *basic function*. A **basic function** is the simplest function of its type. In this case, $g(x) = x$ is the simplest linear function. It is in the form $f(x) = ax + b$, where $a = 1$ and $b = 0$.

You can write the given functions $c(x)$ and $d(x)$ in terms of the basic function $g(x)$. For example, because $g(x) = x$, you can substitute $g(x)$ for x in the equation for $c(x)$, as shown.

$$c(x) = (x) + 3$$
$$\downarrow$$
$$c(x) = g(x) + 3$$

Just replace x with $g(x)$.

1. Write the function $d(x)$ in terms of the basic function $g(x)$.

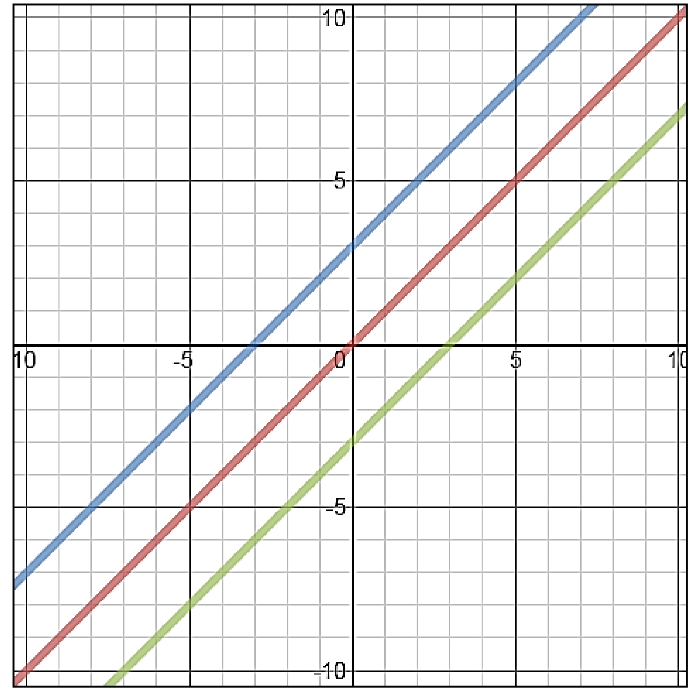
   $d(x) =$ $g(x) - 3$

2. Describe the operation performed on the basic function $g(x)$ to result in each of the equations for $c(x)$ and $d(x)$.

   For $c(x)$, add 3 to $g(x)$.
   For $d(x)$, we subtract 3 from $g(x)$.

3. Use Desmos.com to graph each function: *g*(x), *c*(*x*) and *d*(*x*).
Graph and label your functions

1. $g(x) = x$
2. $c(x) = x + 3$
3. $d(x) = x - 3$



4. Compare the y-intercepts of the graphs of *c*(*x*) and *d*(*x*) to the y-intercept of the basic function *g*(*x*). What do you notice?

For *c*(*x*), the *y*-intercept moves *g*(*x*) UP 3 units.
For *d*(*x*), the *y*-intercept moves *g*(*x*) DOWN 3 units.

5. Write the *y*-value of each ordered pair for the three given functions.

$g(x) = x$	$c(x) = (x) + 3$	$d(x) = (x) - 3$
(−2, -2)	(−2, 1)	(−2, -5)
(−1, -1)	(−1, 2)	(−1, -4)
(0, 0)	(0, 3)	(0, -3)
(1, 1)	(1, 4)	(1, -2)
(2, 2)	(2, 5)	(2, -1)

6. Use the table to compare the ordered pairs of the graphs of $c(x)$ and $d(x)$ to the ordered pairs of the graph of the basic function $g(x)$. What do you notice?

The *x*-coordinates never change.
The *y*-coordinate of $c(x)$ = the *y*-coordinate of $g(x)$ plus 3.
The *y*-coordinate of $d(x)$ = the *y*-coordinate of $g(x)$ minus 3.

A vertical translation is a type of transformation that shifts the entire graph **UP** or **DOWN**.
A vertical translation ***affects the y-coordinate*** of each point on the graph.

A vertical shift occurs when a number is added to or subtracted from the whole basic function!

Now, let's consider the three exponential functions shown.

- $h(x) = 2^x$
- $s(x) = (2^x) + 3$
- $t(x) = (2^x) - 3$

In this case, $h(x) = 2^x$ is the basic function because it is the simplest exponential function with a base of 2. It is in the form $f(x) = a \cdot b^x$, where $a = 1$ and $b = 2$.

8. Write the functions $s(x)$ and $t(x)$ in terms of the basic function $h(x)$. Then, describe the operation performed on the basic function $h(x)$ to result in each of the equations for $s(x)$ and $t(x)$.

$s(x) =$ $h(x) + 3$

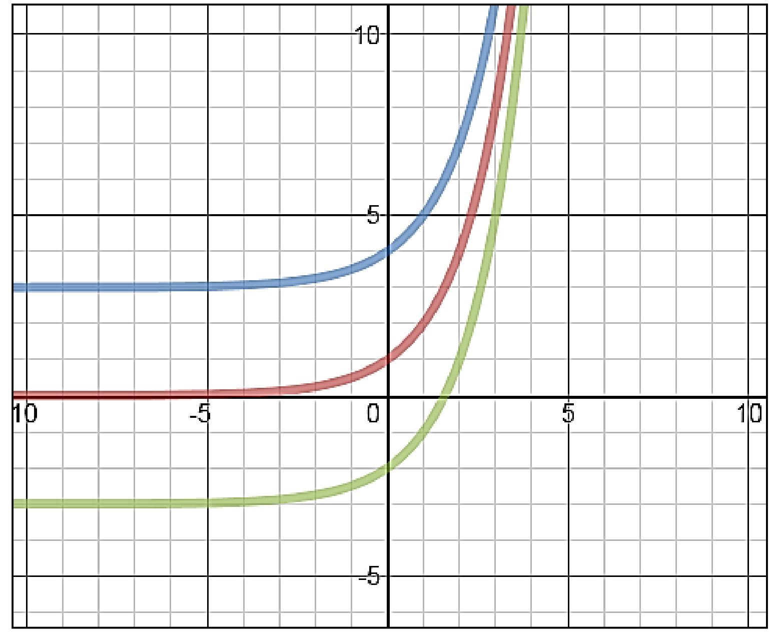
$t(x) =$ $h(x) - 3$

Just replace 2^x with $h(x)$.

For $s(x)$, add 3 to $h(x)$.
For $t(x)$, we subtract 3 from $h(x)$.

9. Use Desmos.com to graph each function: *h*(x), s(*x*), and *t*(*x*).
Label your functions.

1. $h(x) = 2^x$
2. $s(x) = \left(2^x\right) + 3$
3. $t(x) = \left(2^x\right) - 3$



10. Compare the y-intercepts of the graphs of *s*(*x*) and *t*(*x*) to the y-intercept of the basic function *h*(*x*). What do you notice? Are the results the same as when you compared the graphs of the linear functions in Question 4?

For *s*(*x*), the *y*-intercept moves *h*(*x*) UP 3 units.
For *t*(*x*), the *y*-intercept moves *h*(*x*) DOWN 3 units.
Yes, the results are the same.

11. Write the *y*-value of each ordered pair for the three given functions.

$h(x) = 2^x$	$s(x) = (2^x) + 3$	$t(x) = (2^x) - 3$
$(-2, \frac{1}{4}$ or 0.25$)$	$(-2, \frac{13}{4}$ or 3.25$)$	$(-2, -\frac{11}{4}$ or -2.75$)$
$(-1, \frac{1}{2}$ or 0.5$)$	$(-1, \frac{7}{2}$ or 3.5$)$	$(-1, -\frac{5}{2}$ or -2.5$)$
(0, 1)	(0, 4)	(0, -2)
(1, 2)	(1, 5)	(1, -1)
(2, 4)	(2, 7)	(2, 1)

12. Use the table to compare the ordered pairs of the graphs of $s(x)$ and $t(x)$ to the ordered pairs of the graph of the basic function $h(x)$. What do you notice? Are the results the same as when you compared the y-values for the linear functions in Question 6?

The x-coordinates never change.
The y-coordinate of $s(x)$ = the y-coordinate of $h(x)$ plus 3.
The y-coordinate of $t(x)$ = the y-coordinate of $h(x)$ minus 3.
Yes, the results are the same.

13. Explain how you know that the graphs of $s(x)$ and $t(x)$ are vertical translations of the graph of $h(x)$.

The x-coordinates stay the same.
The graph of $s(x)$ moves $h(x)$ UP 3 units.
The graph of $t(x)$ moves $h(x)$ DOWN 3 units.