Let the Transformations Begin!

5.3

Translations of Linear and Exponential Functions

LEARNING GOALS

In this lesson, you will:

- Translate linear and exponential functions vertically.
- Translate linear and exponential functions horizontally.

KEY TERMS

- basic function
- transformation
- vertical translation
- coordinate notation
- argument of a function
- horizontal translation

Skip to Page 319

PROBLEM 2 Horizontal Translations

Consider the three exponential functions shown, where $h(x) = 2^x$ is the basic function.

- $h(x) = 2^x$
- $v(x) = 2^{(x+3)}$
- $w(x) = 2^{(x-3)}$

Also, known as the "parent function"

Simply, add or subtract 3 from *x*.

In Problem 1 *Vertical Translations*, the operations that produced the vertical translations were performed on the function $h(x)$. That is, 3 was added to $h(x)$ and 3 was subtracted from $h(x)$. In this problem, the operations are performed on *x*, which is the *argument* of the function. The **argument of a function** is the *variable* on which the function operates. So, in this case, 3 is added to *x* and 3 is subtracted from *x*.

You can write the given functions $v(x)$ and $w(x)$ in terms of the basic function $h(x)$. To write $v(x)$ in terms of $h(x)$, you just substitute $x + 3$ into the argument for $h(x)$, as shown.

$$h(x) = 2^x$$

$$v(x) = h(x + 3) = 2^{(x+3)}$$

Replace *x* with *x* + 3.

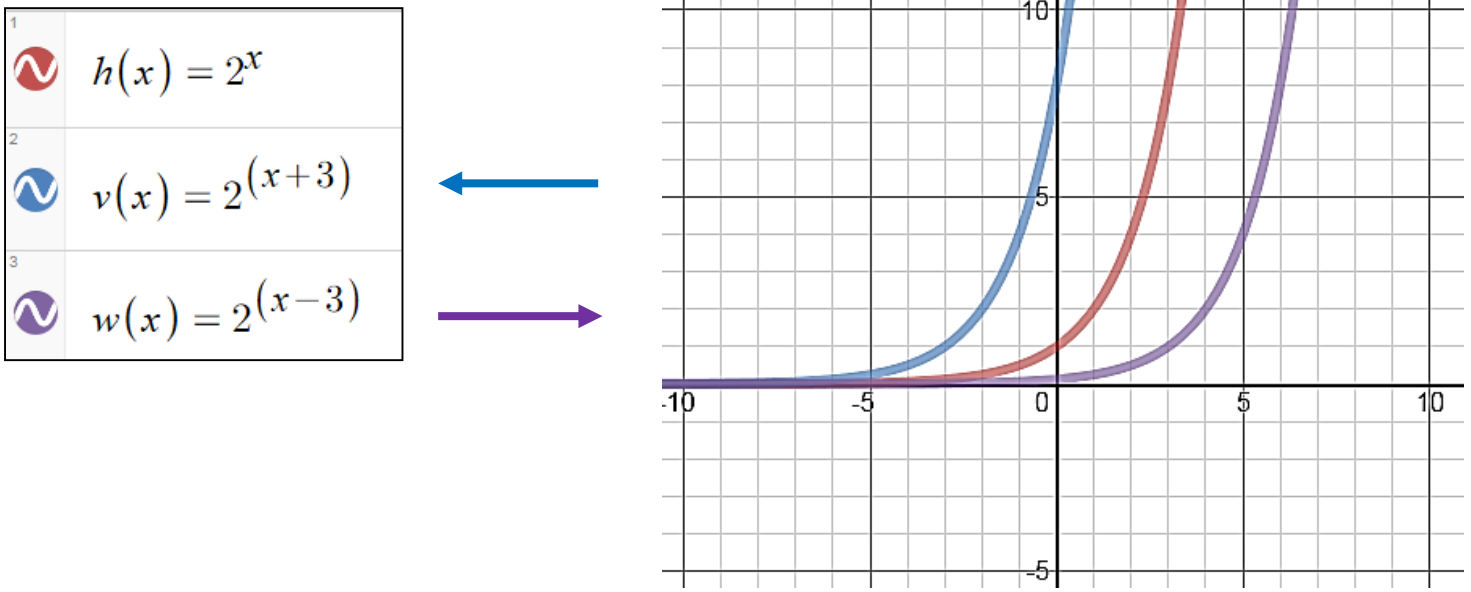
So, $x + 3$ replaces the variable *x* in the function $h(x) = 2^x$.

1. Write the function $w(x)$ in terms of the basic function $h(x)$.

$$w(x) = h(x-3) = 2^{(x-3)}$$

Replace *x* with *x* – 3.

2. Use Desmos.com to graph each function: $h(x)$, $v(x)$ and $w(x)$. Then, sketch the graph and label each function.



3. Compare the graphs of v(x) and w(x) to the graph of the basic function. What do you notice?

This is tricky!!! Look carefully.

The graph of $v(x)$ shifts to the LEFT 3 units.

The graph of $w(x)$ shifts to the RIGHT 3 units.

4. Write the x-value of each ordered pair for the three given functions. You can use your graphing calculator to determine the x-values.

Use the graphs in Desmos.com to find the x-values.

$h(x) = 2^x$	$v(x) = 2^{(x+3)}$	$w(x) = 2^{(x-3)}$
(-2, $\frac{1}{4}$)	(-5, $\frac{1}{4}$)	(1, $\frac{1}{4}$)
(-1, $\frac{1}{2}$)	(-4, $\frac{1}{2}$)	(2, $\frac{1}{2}$)
(0, 1)	(-3, 1)	(3, 1)
(1, 2)	(-2, 2)	(4, 2)
(2, 4)	(-1, 4)	(5, 4)

5. Use the table to compare the ordered pairs of the graphs of $v(x)$ and $w(x)$ to the ordered pairs of the graph of the basic function $h(x)$. What do you notice?

The y-coordinates stay the same.
The x-coordinate of $v(x)$ = the x-coordinate of $h(x)$ minus 3.
The x-coordinate of $w(x)$ = the x-coordinate of $h(x)$ plus 3.

A horizontal translation of a graph is a shift of the entire graph **LEFT** or **RIGHT**. A horizontal translation ***affects the x-coordinate*** of each point on the graph.

Graphing Horizontal and Vertical Translations of Linear and Exponential Functions

	Function Form	Type of Translation	Description of Translation
Linear Functions	$f(x) = x + b$	Vertical translation	UP b units
	$f(x) = x - b$	Vertical translation	DOWN b units
	$f(x) = (x + b)$	Horizontal translation	LEFT b units
	$f(x) = (x - b)$	Horizontal translation	RIGHT b units
Exponential Functions	$f(x) = b^x + k$	Vertical translation	UP k units
	$f(x) = b^x - k$	Vertical translation	DOWN k units
	$f(x) = b^{(x + c)}$	Horizontal translation	LEFT c units
	$f(x) = b^{(x - c)}$	Horizontal translation	RIGHT c units