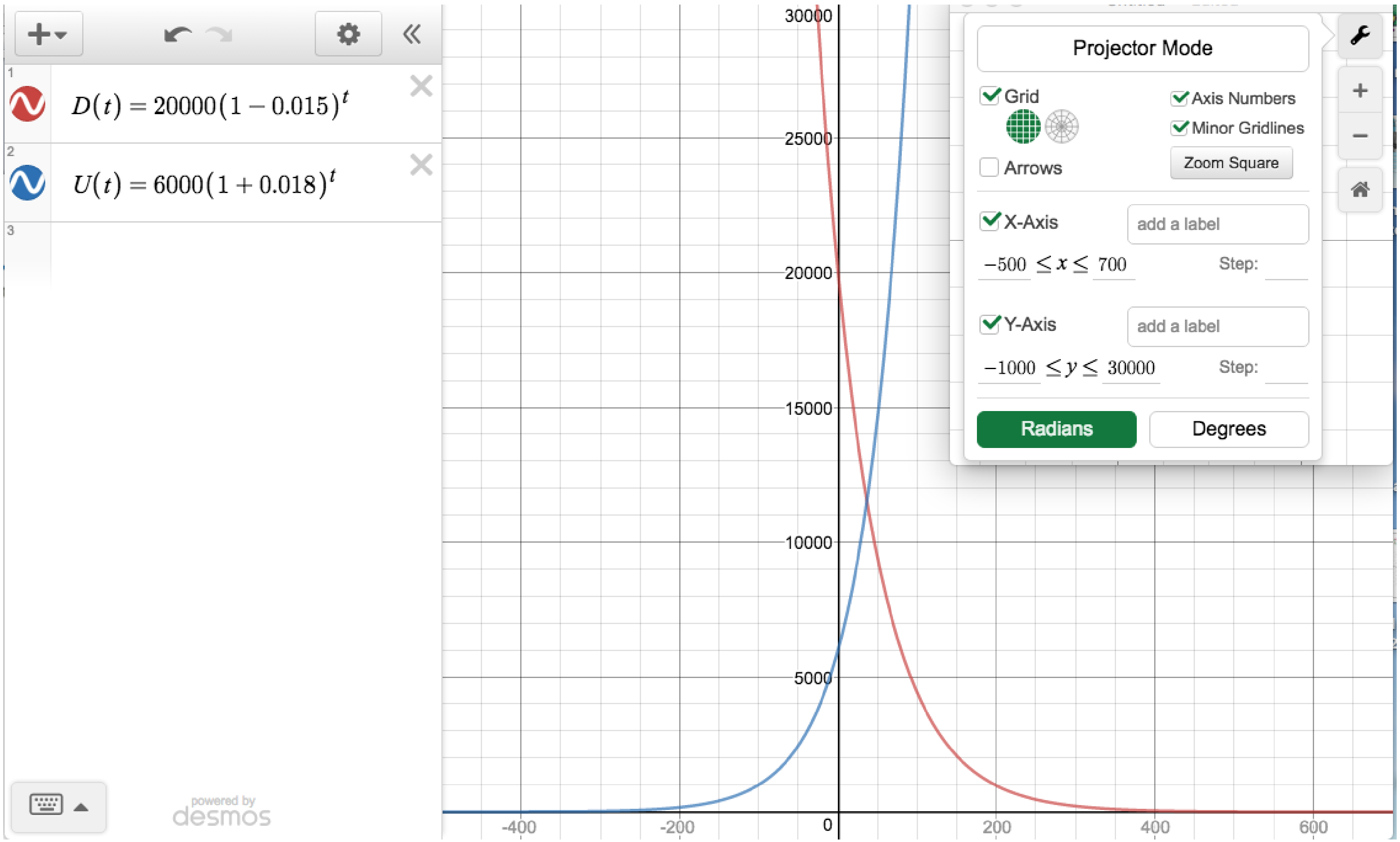
PROBLEM 2 Graphing, Finally!

Let's examine the properties of the graphs of the functions for Downtown and Uptown. Here are the functions again:

Go to Desmos.com.

Downtown: $D(t) = 20{,}000(1 - 0.015)^t$ Uptown: $U(t) = 6000(1 + 0.018)^t$



2. Let's analyze the y-intercepts of each function.

a. Identify the y-intercepts.

Downtown: y-intercept = 20,000
Uptown: y-intercept = 6,000

b. Interpret the meaning of the y-intercept in terms of this problem situation.

The y-intercept is your starting point. It represents the population before any changes occur.

c. Describe how you can determine the y-intercept of each function using just the formula for population increase or decrease.

$D(t) = \underline{20000}(1 - 0.015)^t$ and $U(t) = \underline{6000}(1 + 0.018)^t$ are exponential functions.

Exponential functions are written in the form:

$f(x) = a \cdot b^x$, where a = y-intercept and b = rate of change (or common ratio)

Use Desmos.com

3. Use a graphing calculator to answer each question. Describe your strategy.

a. How long will it take for Downtown's population to be half of what it is now?

Downtown's population = 20,000
20,000/2 = 10,000. So, graph $y = 10000$.
The two graphs intersect when $45 < x < 46$.
So, Downtown's population will be half of what it is now in 45 to 46 years.

b. How long will it take for Uptown's population to double from what it is now?

Uptown's population = 6,000
$6{,}000 \times 2 = 12{,}000$. So, graph $y = 12000$.
The two graphs intersect when $38 < x < 39$.
So, Uptown's population will double in 38 to 39 years.

c. How many years from now will the populations of Downtown and Uptown be equal? Determine the approximate populations.

Recall:
x-values (IQ) = time or the # of years from now
y-values (DQ) = population

Find the point-of-intersection for the two graphs.
Between 36 and 37 years from now, both Downtown and Uptown will have approximately 11,500 people.

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Each population function you graphed has a *horizontal asymptote*. A **horizontal asymptote** is a horizontal line that a function gets closer and closer to, but never intersects.

6. Write the equation for the horizontal asymptote of each population function.

$y = 0$

Does horizontal mean ↕ or ↔?

7. Does the horizontal asymptote make sense in terms of this problem situation? Explain your reasoning.

Not really. If the population is decreasing, it can eventually reach 0. Strictly talking numbers, the *y*-values get closer, but never reach "0".

8. Identify the domain and range of each function.

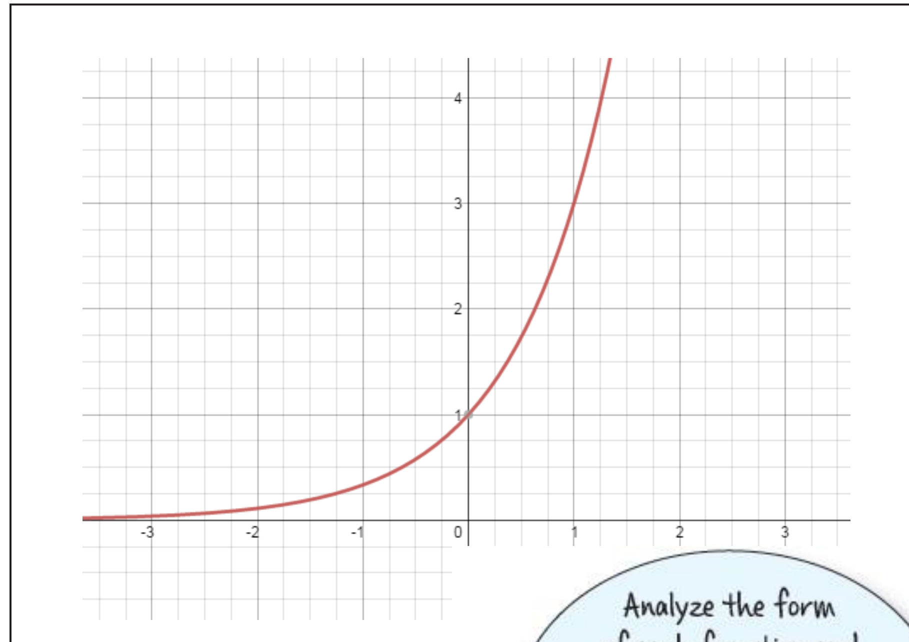
Domain = all real numbers (you can plug in any *x*-value)
Range = all real numbers greater than zero ($y > 0$)

PROBLEM 3 The Multiple Representations of Exponentials

1. Complete the table and sketch a graph for each exponential function of the form $f(x) = ab^x$. Then determine the x-intercept(s), y-intercept, asymptote, domain, range, and interval(s) of increase/decrease.

a. $f(x) = 3^x$

x	$f(x)$
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9
3	27



x-intercept(s): None

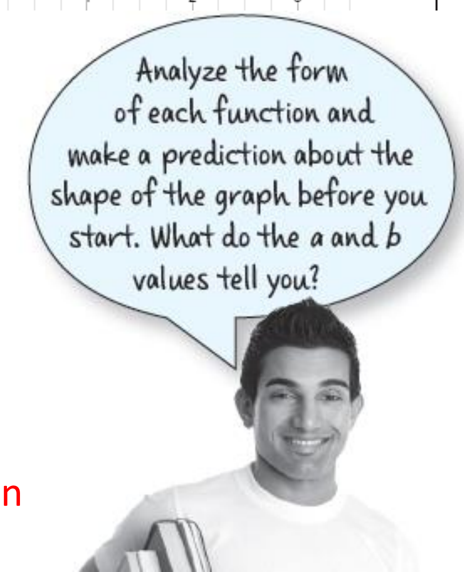
y-intercept: (0, 1)

asymptote: $y = 0$

domain: All Real #'s

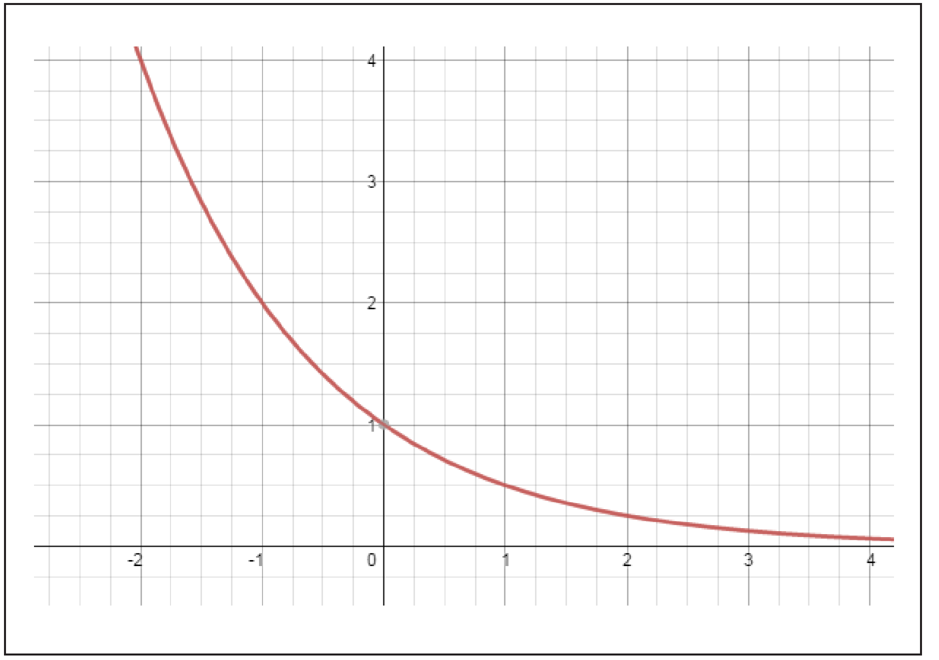
range: $y > 0$

interval(s) of increase/decrease: Increasing over the entire domain



b. $g(x) = \left(\frac{1}{2}\right)^x$

x	$g(x)$
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$
3	$\frac{1}{8}$



x-intercept(s): None

y-intercept: (0, 1)

asymptote: y = 0

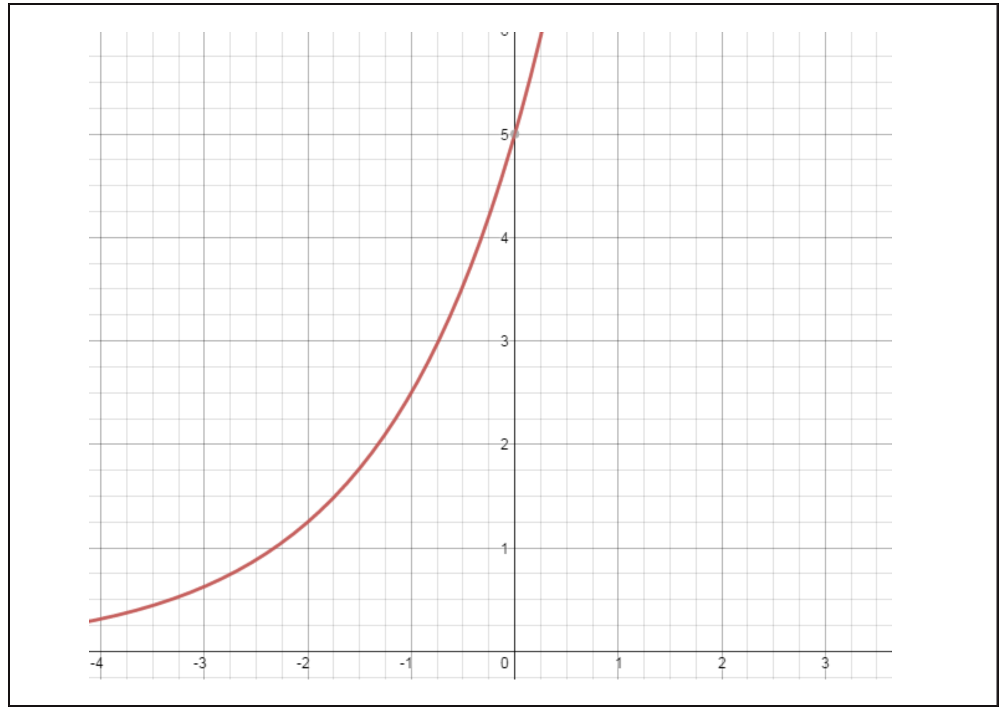
domain: All Real #'s

range: y > 0

interval(s) of increase/decrease: Decreasing over the entire domain

c. $k(x) = 5 \cdot 2^x$

x	$k(x)$
-2	$\frac{5}{4}$
-1	$\frac{5}{2}$
0	5
1	10
2	20
3	40



x-intercept(s): None

y-intercept: (0, 5)

asymptote: y = 0

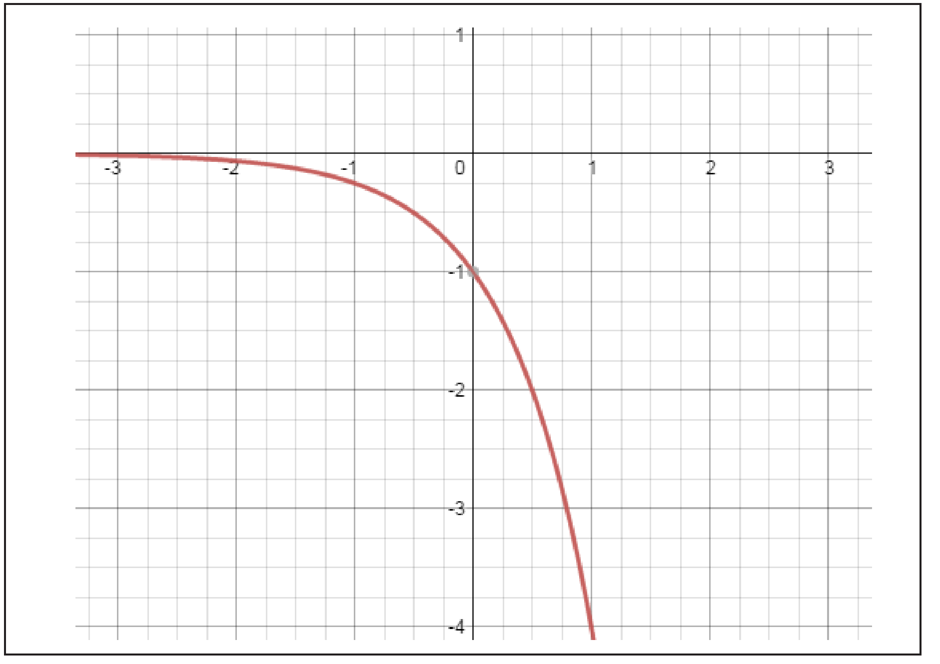
domain: All Real #'s

range: y > 0

interval(s) of increase/decrease: Increasing for entire domain

d. $p(x) = -4^x$

x	$p(x)$
-2	$-\frac{1}{16}$
-1	$-\frac{1}{4}$
0	-1
1	-4
2	-16
3	-64



x-intercept(s): None

y-intercept: (0, -1)

asymptote: y = 0

domain: All Real #'s

range: y < 0

interval(s) of increase/decrease: Decreasing for entire domain

2. Write an exponential equation of the form $y = ab^x$ for each. Explain your reasoning.

a.

x	y
0	1
1	4
2	16
3	64

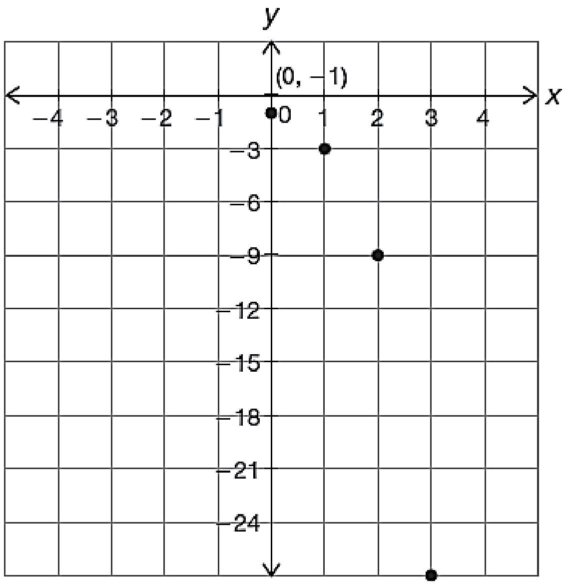
From the table, we can the starting point or *y*-intercept is (0,1). So, a = 1.

The common ratio is 4.

Therefore the equation is $y = 1(4)^x$

$or\ \ y = 4^x$

b.



The y intercept is (0,-1) so a = -1

The common ratio is 3.

Therefore the equation is $y = -1(3)^x$

$or\ \ y = -3^x$

3. Given a function of the form $f(x) = ab^x$.

a. What does the *a*-value tell you?

a = *y*-intercept

b. What does the *b*-value tell you?

b = the common ratio (*r*) between any two points