5.2

Downtown and Uptown

Graphs of Exponential Functions

LEARNING GOALS

In this lesson, you will:

- Solve exponential functions using the intersection of graphs.
- Analyze asymptotes of exponential functions and their meanings in context.
- Identify the domain and range of exponential functions.
- Analyze and graph decreasing exponential functions.
- Compare graphs of linear and exponential functions through intercepts, asymptotes, and end behavior.

KEY TERM

- horizontal asymptote

PROBLEM 1 Downtown and Uptown

At this moment, the population of Downtown is 20,000, and the population of Uptown is 6000. But over many years, people have been moving away from Downtown at a rate of 1.5% every year. At the same time, Uptown's population has been growing at a rate of 1.8% each year.

1. What are the independent and dependent quantities in each situation?

   Independent Quantity (IQ): time (years) Dependent Quantity (DQ): population

2. Which city's population can be represented as an increasing function, and which can be represented as a decreasing function?

   Uptown: Increasing function Downtown: Decreasing function

Add r when the population increases.

Let's analyze the population growth of Uptown. In 1 year from now, the population of Uptown will be

~~6000 + 6000(0.018) = 6108~~.

The population will be 6108 people in Uptown 1 year from now.

$P(t) = P(1 + r)^t$
$P(t) = 6000(1.018)^t$
$P(1) = 6000(1.018)^1 = 6{,}108$

3. Write and simplify an expression that represents the population of Uptown:

   a. 2 years from now.

   $P(2) = 6000(1.018)^2 = 6217.944 \approx 6218$
   There will be about 6,218 people 2 years from now.

   b. 3 years from now.

   $P(3) = 6000(1.018)^3 = 6329.866992 \approx 6330$
   There will be about 6,330 people 3 years from now.

4. How can you tell that this function is an exponential function? Explain your reasoning.

Uptown's population increasing at a variable rate. The rate of growth is not constant.

1st year: 6108 – 6000 = 108 more people
2nd year: 6218 – 6108 = 110 more people
3rd year: 6330 – 6218 = 112 more people

You can use the formula for compound interest to determine the function for Uptown's increasing population. Recall that the formula for compound interest is $P(t) = P(1 + r)^t$, where $P(t)$ represents the amount in the account after a certain amount of time in years, r is the interest rate written as a decimal, and t is the time in years.

Compound interest and populations grow exponentially so we can use the same formula for both.

P(t) = population after *t* years

P = population

r = growth rate (increasing/decreasing)

t = number of years

5. Skip this question.

Now let's analyze the population decline of Downtown.

6. Write and simplify an expression that represents the population of Downtown. The first one has been done for you.

$P(t) = P(1 - r)^t$

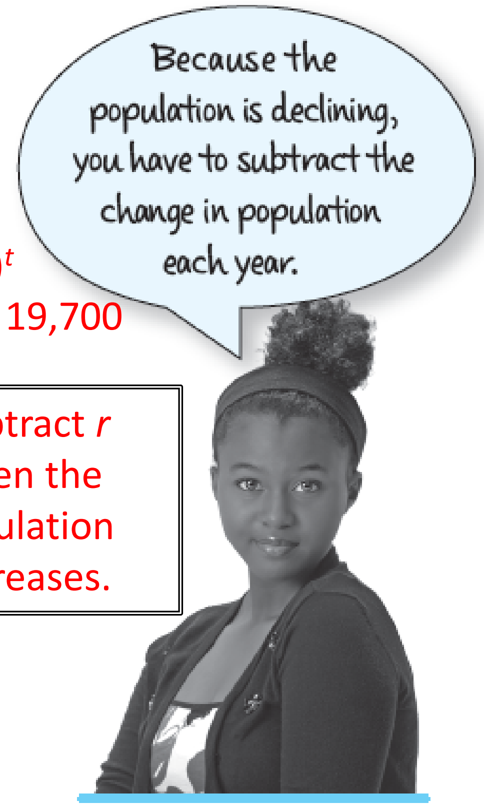
$P(t) = 20000(1 - 0.015)^t$

$P(1) = 20000(0.985)^1 = 19{,}700$

a. 1 year from now.

$\cancel{20{,}000 - 20{,}000(0.015) = 19{,}700}$

The population of Downtown will be 19,700 people 1 year from now.



Subtract r when the population decreases.

b. 2 years from now.

$P(2) = 20000(0.985)^2 = 19404.5 \approx 19405$

There will be about 19,405 people 2 years from now.

c. 3 years from now.

$P(3) = 20000(0.985)^3 = 19113.4325 \approx 19113$

There will be about 19,113 people 3 years from now.

7. Skip this question.

8. Skip this question.

9. Think about each function as representing a sequence.
What is the common ratio in simplest form, or the number that is multiplied each time to get the next term, in each sequence?

For Uptown, the common ratio, or *r,* is 1 + 0.018 or 1.018.
For Downtown, the common ratio, or *r*, is 1 – 0.18 or 0.985.

10. Explain how the common ratios determine whether the exponential functions for the change in population are increasing or decreasing.

If $r > 1$, the exponential function is increasing.
If $0 < r < 1$, the exponential function is decreasing.