

Arithmetic and Geometric Sequences - Recursive Formulas

When you want to find the *next* term in an arithmetic or geometric sequence...

Use a recursive formula.

NOW-NEXT Formula! You have a number now, what's the next number?

$$a_n = a_{n-1} + d$$

Example:

Determine the next 3 terms of the sequence 30, 70, 110, ...

Find the common difference:

$$d = \text{common difference} = 2^{\text{nd}} \text{ term} - 1^{\text{st}} \text{ term} = 70 - 30 = 40$$

Use the recursive formula to solve:

Given the first 3 terms, find the 4th, 5th, and 6th terms.

$$a_4 = 110 + 40 = 150$$

$$a_5 = 150 + 40 = 190$$

$$a_6 = 190 + 40 = 230$$

Determine the next terms in the given arithmetic sequence using the recursive formula.

1. Determine the next 2 terms of the sequence 16, 30, 44, 58, ...

$$d = 30 - 16 = 14$$

Given 4 terms,

$$a_5 = 58 + 14 = 72$$

$$a_6 = 72 + 14 = 86$$

2. Determine the next 3 terms of the sequence -68, -83, -98, ...

$$d = -83 - (-68) = -83 + 68 = -15$$

Given 3 terms,

$$a_4 = -98 + (-15) = -113$$

$$a_5 = -113 + (-15) = -128$$

$$a_6 = -128 + (-15) = -143$$

3. Determine the next 4 terms of the sequence 7.3, 9.4, 11.5, ...

$$d = 9.4 - 7.3 = 2.1$$

Given 3 terms,

$$a_4 = 11.5 + 2.1 = 13.6$$

$$a_5 = 13.6 + 2.1 = 15.7$$

$$a_6 = 15.7 + 2.1 = 17.8$$

$$a_7 = 17.8 + 2.1 = 19.9$$

4. Determine the next 2 terms of the sequence $\frac{1}{2}$, 1, $\frac{3}{2}$, 2, ...

$$d = 1 - \frac{1}{2} = \frac{1}{2}$$

Given 4 terms,

$$a_5 = 2 + \frac{1}{2} = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}$$

$$a_6 = \frac{5}{2} + \frac{1}{2} = \frac{6}{2} \text{ or } 3$$

Determine the next terms in the given geometric sequence using the recursive formula.

$$g_n = g_{n-1} \cdot r$$

Example:

Determine the next 3 terms in the sequence 100, -50, 25, ...

Find the common ratio:

$$r = \text{common ratio} = \frac{\text{2nd term}}{\text{1st term}} = \frac{-50}{100} = -0.5$$

Use the recursive formula to solve:

Given the first 3 terms, find the 4th, 5th, and 6th terms.

$$g_4 = 25 \cdot (-0.5) = -12.5$$

$$g_5 = -12.5 \cdot (-0.5) = 6.25$$

$$g_6 = 6.25 \cdot (-0.5) = -3.125$$

5. Determine the next 2 terms in the sequence 4, 8, 16, 32, ...

$$r = \frac{8}{4} = 2$$

Given 4 terms,

$$g_5 = 32 \cdot 2 = 64$$

$$g_6 = 64 \cdot 2 = 128$$

6. Determine the next 3 terms in the sequence -5, 20, -80, ...

$$r = \frac{20}{-5} = -4$$

Given 3 terms,

$$g_4 = -80 \cdot (-4) = 320$$

$$g_5 = 320 \cdot (-4) = -1280$$

$$g_6 = -1280 \cdot (-4) = 5120$$

7. Determine the next 4 terms in the sequence 2, -6, 18, ...

$$r = \frac{-6}{2} = -3$$

Given 3 terms,

$$g_4 = 18 \cdot (-3) = -54$$

$$g_5 = -54 \cdot (-3) = 162$$

$$g_6 = 162 \cdot (-3) = -486$$

$$g_7 = -486 \cdot (-3) = 1458$$

8. Determine the next 2 terms in the sequence 3, 1.5, 0.75, ...

$$r = \frac{1.5}{3} = 0.5$$

Given 3 terms,

$$g_4 = 0.75 \cdot 0.5 = 0.375$$

$$g_5 = 0.375 \cdot 0.5 = 0.1875$$