

Arithmetic and Geometric Sequences - Explicit Formulas

When you want to find the n th term in an arithmetic or geometric sequence?

Use an explicit formula.

Determine each unknown term in the given arithmetic sequence using the explicit formula.

$$a_n = a_1 + d(n-1)$$

Example:

Determine the 20th term of the sequence 1, 4, 7, ...

Define your variables:

$$n = \text{term number} = 20$$

$$a_n = \text{nth term} = a_{20}$$

$$a_1 = \text{1st term} = 1$$

$$d = \text{common difference} = \text{2nd term} - \text{1st term} = 4 - 1 = 3$$

Use the explicit formula to solve:

$$a_{20} = 1 + 3(20 - 1)$$

$$a_{20} = 1 + 3(19)$$

$$a_{20} = 1 + 57$$

$$a_{20} = 58$$

1. Determine the 30th term of the sequence
-10, -15, -20, ...

$$\begin{aligned} n &= 30 & a_{30} &= -10 + (-5)(30-1) \\ a_n &= a_{30} & a_{30} &= -10 + (-5)(29) \\ a_1 &= -10 & a_{30} &= -10 - 145 \\ d &= -15 - (-10) & a_{30} &= -155 \\ d &= -15 + 10 & & \\ d &= -5 & & \end{aligned}$$

3. Determine the 42nd term of the sequence
12.25, 14.50, 16.75, ...

$$\begin{aligned} n &= 42 & a_{42} &= 12.25 + 2.25(42-1) \\ a_n &= a_{42} & a_{42} &= 12.25 + 2.25(41) \\ a_1 &= 12.25 & a_{42} &= 12.25 + 92.25 \\ d &= 14.50 - 12.25 & a_{42} &= 104.5 \\ d &= 2.25 & & \end{aligned}$$

2. Determine the 50th term of the sequence
100, 92, 84, ...

$$\begin{aligned} n &= 50 & a_{50} &= 100 + (-8)(50-1) \\ a_n &= a_{50} & a_{50} &= 100 + (-8)(49) \\ a_1 &= 100 & a_{50} &= 100 - 392 \\ d &= 92 - 100 & a_{50} &= -292 \\ d &= -8 & & \end{aligned}$$

4. Determine the 25th term of the sequence
3.3, 4.4, 5.5, ...

$$\begin{aligned} n &= 25 & a_{25} &= 3.3 + 1.1(25-1) \\ a_n &= a_{25} & a_{25} &= 3.3 + 1.1(24) \\ a_1 &= 3.3 & a_{25} &= 3.3 + 26.4 \\ d &= 4.4 - 3.3 & a_{25} &= 29.7 \\ d &= 1.1 & & \end{aligned}$$

Determine each unknown term in the given geometric sequence using the explicit formula. Round to the nearest 100th if necessary.

$$g_n = g_1 \cdot r^{n-1}$$

Example:

Determine the 15th term of the sequence 0.125, -0.250, 0.500, ...

Define your variables:

$$n = \text{term number} = 15$$

$$g_n = \text{nth term} = g_{15}$$

$$g_1 = \text{1st term} = 0.125$$

$$r = \text{common ratio} = \frac{\text{2nd term}}{\text{1st term}} = \frac{-0.250}{0.125} = -2$$

Use the explicit formula to solve:

$$g_{15} = 0.125 \cdot (-2)^{15-1}$$

$$g_{15} = 0.125 \cdot (-2)^{14}$$

$$g_{15} = 0.125 \cdot 16384$$

$$g_{15} = 2048$$

5. Determine the 10th term of the sequence
3, 6, 12, ...

$$\begin{aligned} n &= 10 & g_{10} &= 3 \cdot 2^{10-1} \\ g_n &= g_{10} & g_{10} &= 3 \cdot 2^9 \\ g_1 &= 3 & g_{10} &= 3 \cdot 512 \\ r &= \frac{6}{3} = 2 & g_{10} &= 1536 \end{aligned}$$

6. Determine the 15th term of the sequence
1, -2, 4, ...

$$\begin{aligned} n &= 15 & g_{15} &= 1 \cdot (-2)^{15-1} \\ g_n &= g_{15} & g_{15} &= 1 \cdot (-2)^{14} \\ g_1 &= 1 & g_{15} &= 1 \cdot 16384 \\ r &= \frac{-2}{1} = -2 & g_{15} &= 16384 \end{aligned}$$

7. Determine the 18th term of the sequence
3, 9, 27, ...

$$\begin{aligned} n &= 18 & g_{18} &= 3 \cdot 3^{18-1} \\ g_n &= g_{18} & g_{18} &= 3 \cdot 3^{17} \\ g_1 &= 3 & g_{18} &= 3 \cdot 129140163 \\ r &= \frac{9}{3} = 3 & g_{18} &= 387420489 \end{aligned}$$

8. Determine the 12th term of the sequence
4, 5, 6.25, ...

$$\begin{aligned} n &= 12 & g_{12} &= 4 \cdot 1.25^{12-1} \\ g_n &= g_{12} & g_{12} &= 4 \cdot 1.25^{11} \\ g_1 &= 4 & g_{12} &= 4 \cdot 11.64153218 \\ r &= \frac{5}{4} = 1.25 & g_{12} &\approx 46.57 \end{aligned}$$

use decimals
like the
sequence