Page 235

The Power of Algebra Is a Curious Thing Using Formulas to Determine Terms of a Sequence

4.3

LEARNING GOALS

In this lesson, you will:

- Write an explicit formula for arithmetic and geometric formulas.
- Write a recursive formula for arithmetic and geometric formulas.
- Use formulas to determine unknown terms of a sequence.

KEY TERMS

- index
- explicit formula
- recursive formula

PROBLEM 1 Can I Get a Formula?

Page 236

While a common ratio or a common difference can help you determine the next term in a sequence, how can they help you determine the thousandth term of a sequence? The ten-thousandth term of a sequence? Consider the sequence represented in the given problem scenario.



 Rico owns a sporting goods store. He has agreed to donate \$125 to the Centipede Valley High School baseball team for their equipment fund. In addition, he will donate \$18 for every home run the Centipedes hit during the season. The sequence shown represents the possible dollar amounts that Rico could donate for the season.

125, 143, 161, 179, . . .

a. Identify the sequence type. Describe how you know.

143 – 125 = 18 61 – 143 = 18 You add 18 each time so it's arithmetic.

b. Determine the common ratio or common difference for the given sequence.

The **common difference**, d = 18.

c. Complete the table of values. Use the number of home runs the Centipedes could hit to identify the term number, and the total dollar amount Rico could donate to the baseball team.

Notice that the 1st term in this sequence is the amount Rico donates if the team hits 0 home runs.



Number of Home Runs	Term Number (<i>n</i>)	Donation Amount (dollars)
0	1	125 + 18
1	2	143 + 18
2	3	161
3	4	179
4	5	197
5	6	215
6	7	233
7	8	251
8	9	269
9	10	287

Starting point

This process is so tedious. There's got to be an easier way!

- d. Explain how you can calculate the tenth term based on the ninth term.
 Add 18 to the 9th term
- e. Determine the 20th term. Explain your calculation. 20th term = 467 How many more times would you need to add 18 to calculate the 20th term?
- f. Is there a way to calculate the 20th term without first calculating the 19th term? If so, describe the strategy.

Starting point = \$125 ROC (rate of change) = \$18/home run Let x = # of home runs Use slope-intercept form: 18x + 125 \bigcirc

g. Describe a strategy to calculate the 93rd term.

Let's try using slope-intercept form!

Looking back at our table on page 236, the 93rd term means how many home runs?

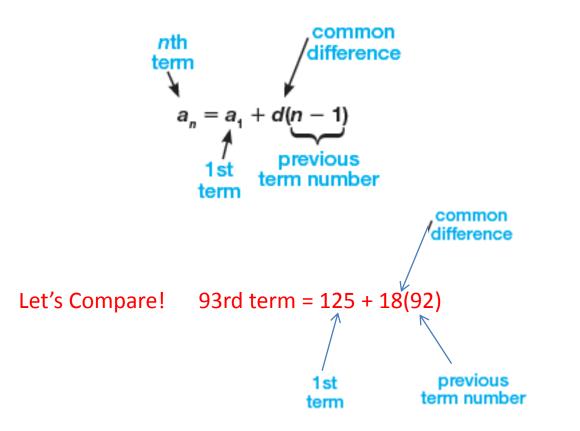
Let *x* = 92

 93^{rd} term = 18(92) + 125 = \$1781

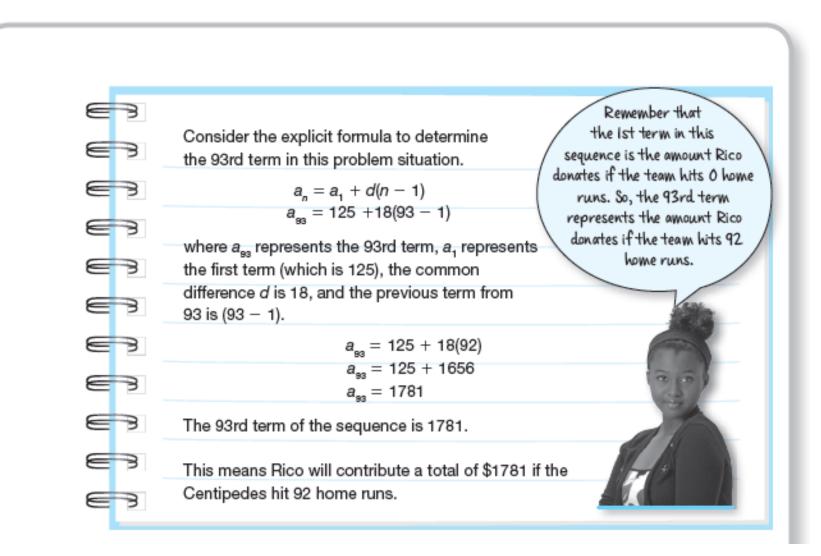
We've created an explicit formula!

Go to the Bottom of Page 238.

An **explicit formula** of a sequence is a formula for calculating the value of each term of a sequence using the term's position in the sequence. The explicit formula for determining the *n*th term of an arithmetic sequence is:



Let's look at the problem we just solved.



$$a_n = a_1 + d(n-1)$$

- 3. Use the explicit formula to determine the amount of money Rico will contribute if the Centipedes hit:
 - a. 35 home runs. (36th term!) b. 48 home runs.
- $a_{36} = 125 + 18(36 1)$ $a_{36} = 125 + 18(35)$ $a_{36} = \$755$
 - c. 86 home runs.

 $a_{49} = 125 + 18(49 - 1)$ $a_{49} = 125 + 18(48)$ $a_{49} = \$989$

d. 214 home runs.

 $a_{87} = 125 + 18(87 - 1)$ $a_{87} = 125 + 18(86)$ $a_{87} = \$1673$

 $a_{215} = 125 + 18(215 - 1)$ $a_{215} = 125 + 18(214)$ $a_{215} = \$3977$

4.3 Using Formulas to Determine Terms of a Sequence 😑 239



 $a_n = a_1 + d(n-1) \longrightarrow a_n = 500 + 75(n-1)$

- 4. Rico decides to increase his initial contribution and amount donated per home run hit. He decides to contribute \$500 and will donate \$75.00 for every home run the Centipedes hit. Determine Rico's contribution if the Centipedes hit:
 - a. 11 home runs.
- $a_{12} = 500 + 75(12 1)$ $a_{12} = 500 + 75(11)$ $a_{12} = \$1325$

- b. 26 home runs.
- $\begin{aligned} a_{27} &= 500 + 75(27 1) \\ a_{27} &= 500 + 75(26) \\ a_{27} &= \$2450 \end{aligned}$





Let's Practice!