

4.3

The Power of Algebra Is a Curious Thing

Using Formulas to Determine Terms of a Sequence

LEARNING GOALS

In this lesson, you will:

- Write an explicit formula for arithmetic and geometric formulas.
- Write a recursive formula for arithmetic and geometric formulas.
- Use formulas to determine unknown terms of a sequence.

KEY TERMS

- index
- explicit formula
- recursive formula



While a common ratio or a common difference can help you determine the next term in a sequence, how can they help you determine the thousandth term of a sequence? The ten-thousandth term of a sequence? Consider the sequence represented in the given problem scenario.



1. Rico owns a sporting goods store. He has agreed to donate \$125 to the Centipede Valley High School baseball team for their equipment fund. In addition, he will donate \$18 for every home run the Centipedes hit during the season. The sequence shown represents the possible dollar amounts that Rico could donate for the season.

125, 143, 161, 179, . . .

- a. Identify the sequence type. Describe how you know.

$$143 - 125 = 18$$

$$161 - 143 = 18$$

You add 18 each time so it's arithmetic.

- b. Determine the common ratio or common difference for the given sequence.

The **common difference**, $d = 18$.

- c. Complete the table of values. Use the number of home runs the Centipedes could hit to identify the term number, and the total dollar amount Rico could donate to the baseball team.

Notice that the 1st term in this sequence is the amount Rico donates if the team hits 0 home runs.



Number of Home Runs	Term Number (n)	Donation Amount (dollars)
0	1	125
1	2	143
2	3	161
3	4	179
4	5	197
5	6	215
6	7	233
7	8	251
8	9	269
9	10	287

Starting point

This process is so tedious. There's got to be an easier way!

d. Explain how you can calculate the tenth term based on the ninth term.

Add 18 to the 9th term

e. Determine the 20th term. Explain your calculation.

20th term = 467

How many more times would you need to add 18 to calculate the 20th term?

f. Is there a way to calculate the 20th term without first calculating the 19th term?
If so, describe the strategy.

Starting point = \$125

ROC (rate of change) = \$18/home run

Let x = # of home runs

Use slope-intercept form: $18x + 125$ 😊

g. Describe a strategy to calculate the 93rd term.

Let's try using slope-intercept form!

Looking back at our table on page 236, the 93rd term means how many home runs?

Let $x = 92$

93rd term = $18(92) + 125 = \$1781$

We've created an explicit formula!

Go to the Bottom of Page 238.

An explicit formula of a sequence is a formula for calculating the value of each term of a sequence using the term's position in the sequence. The explicit formula for determining the n th term of an arithmetic sequence is:

$$a_n = a_1 + d(n - 1)$$

nth term points to a_n
1st term points to a_1
common difference points to d
previous term number points to $(n - 1)$

Let's Compare! $93\text{rd term} = 125 + 18(92)$

1st term points to 125
common difference points to 18
previous term number points to 92

Let's look at the problem we just solved.

Consider the explicit formula to determine the 93rd term in this problem situation.

$$a_n = a_1 + d(n - 1)$$
$$a_{93} = 125 + 18(93 - 1)$$

where a_{93} represents the 93rd term, a_1 represents the first term (which is 125), the common difference d is 18, and the previous term from 93 is $(93 - 1)$.

$$a_{93} = 125 + 18(92)$$
$$a_{93} = 125 + 1656$$
$$a_{93} = 1781$$

The 93rd term of the sequence is 1781.

This means Rico will contribute a total of \$1781 if the Centipedes hit 92 home runs.

Remember that the 1st term in this sequence is the amount Rico donates if the team hits 0 home runs. So, the 93rd term represents the amount Rico donates if the team hits 92 home runs.



$$a_n = a_1 + d(n-1)$$

3. Use the explicit formula to determine the amount of money Rico will contribute if the Centipedes hit:

- a. 35 home runs. (*36th term!*) b. 48 home runs.

$$a_{36} = 125 + 18(36 - 1)$$

$$a_{49} = 125 + 18(49 - 1)$$

$$a_{36} = 125 + 18(35)$$

$$a_{49} = 125 + 18(48)$$

$$a_{36} = \$755$$

$$a_{49} = \$989$$

- c. 86 home runs.

- d. 214 home runs.

$$a_{87} = 125 + 18(87 - 1)$$

$$a_{215} = 125 + 18(215 - 1)$$

$$a_{87} = 125 + 18(86)$$

$$a_{215} = 125 + 18(214)$$

$$a_{87} = \$1673$$

$$a_{215} = \$3977$$

Remember, the term number is not the same as the number of home runs!

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$$a_n = a_1 + d(n-1) \longrightarrow a_n = 500 + 75(n-1)$$

4. Rico decides to increase his initial contribution and amount donated per home run hit. He decides to contribute \$500 and will donate \$75.00 for every home run the Centipedes hit. Determine Rico's contribution if the Centipedes hit:

a. 11 home runs.

$$a_{12} = 500 + 75(12-1)$$

$$a_{12} = 500 + 75(11)$$

$$a_{12} = \$1325$$

b. 26 home runs.

$$a_{27} = 500 + 75(27-1)$$

$$a_{27} = 500 + 75(26)$$

$$a_{27} = \$2450$$

~~c. 39 home runs.~~

~~d. 50 home runs.~~

Let's Practice!