

PROBLEM 2 They're Just Out of Control—But That's A Good Thing!



When it comes to bugs, bats, spiders, and—ugh, any other creepy crawlers—finding one in your house is finding *one* too many! Then again, when it comes to cells, the more the better! Animals, plants, fungi, slime, molds, and other living creatures consist of eukaryotic cells. During growth, generally there is a cell called a “mother cell” that divides itself into two “daughter cells.” Each of those daughter cells then divides into two more daughter cells, and so on.

- The sequence shown represents the growth of eukaryotic cells.

1, 2, 4, 8, 16, . . .

- Describe why this sequence is geometric.

$$\frac{2}{1} = 2 \quad \text{and} \quad \frac{4}{2} = 2$$

You multiply by 2 each time.

Notice that the 1st term in this sequence is the total number of cells after 0 divisions (that is, the mother cell).



b. Determine the common ratio for the given sequence.

The **common ratio**, $r = 2$.

c. Complete the table of values. Use the number of cell divisions to identify the term number, and the total number of cells after each division.

Compare the
Number of Cell
Divisions to the
Term Number.
What's the
difference?

Number of Cell Divisions	Term Number (n)	Total Number of Cells
0	1	1
1	2	2
2	3	4
3	4	8
4	5	16
5	6	32
6	7	64
7	8	128
8	9	256
9	10	512

Starting point

Is there
another way
to generate
these
numbers?

Do you notice
any patterns
in the data?

d. Explain how you can calculate the tenth term based on the ninth term.

Multiply the 9th term by 2.

e. Determine the 20th term. Explain your calculation. Keep multiplying by 2!

20th term = 524,288.

f. Is there a way to calculate the 20th term without first calculating the 19th term?
If so, describe the strategy.

Yes. Use the formula 2^x , where x = the # of cell divisions or the # of terms - 1.

Go
back to
the top
of the
page.

b. Determine the common ratio for the given sequence.

The **common ratio**, $r = 2$.

c. Complete the table of values. Use the number of cell divisions to identify the term number, and the total number of cells after each division.

Number of Cell Divisions	Term Number (n)	Total Number of Cells
0	1	$1 = 2^0$
1	2	$2 = 2^1$
2	3	$4 = 2^2$
3	4	$8 = 2^3$
4	5	$16 = 2^4$
5	6	$32 = 2^5$
6	7	$64 = 2^6$
7	8	$128 = 2^7$
8	9	$256 = 2^8$
9	10	$512 = 2^9$

Rewrite the
total # of cells
in exponential
form.

Go to the Bottom of Page 242.

The explicit formula for determining the n th term of a geometric sequence is:

The diagram illustrates the explicit formula for the n th term of a geometric sequence, $g_n = g_1 \cdot r^{n-1}$, and compares it to a specific example, $(1)2^x$. Blue arrows point from descriptive labels to the corresponding parts of the formula and example.

Formula: $g_n = g_1 \cdot r^{n-1}$

- g_n : n th term
- g_1 : 1st term
- r^{n-1} : common ratio (where $n-1$ is the previous term number)

Example: $(1)2^x$

- (1) : 1st term
- 2^x : common ratio (where x is the previous term number)

Let's Compare!

$$g_n = g_1 \cdot r^{n-1}$$

Go to Page 243.

3. Use the explicit formula to determine the total number of cells after:

a. 11 divisions. (12th term!)

$$g_{12} = 1 \cdot 2^{12-1}$$

$$g_{12} = 1 \cdot 2^{11}$$

$$g_{12} = 2^{11}$$

$$g_{12} = 2048$$

~~c. 18 divisions.~~

b. 14 divisions.

$$g_{15} = 1 \cdot 2^{15-1}$$

$$g_{15} = 1 \cdot 2^{14}$$

$$g_{15} = 2^{14}$$

$$g_{15} = 16,384$$

~~d. 22 divisions.~~

$$g_n = g_1 \cdot r^{n-1} \longrightarrow \begin{array}{l} \text{1st Term} = 5 \\ r = 3 \end{array} \longrightarrow g_n = 5 \cdot 3^{n-1}$$

4. Suppose that a scientist has 5 eukaryotic cells in a petri dish. She wonders how the growth pattern would change if each mother cell divided into 3 daughter cells. For this situation, determine the total number of cells in the petri dish after:

a. 4 divisions. (*5th term!*)

$$g_5 = 5 \cdot 3^{5-1}$$

$$g_5 = 5 \cdot 3^4$$

$$g_5 = 5 \cdot 81$$

$$g_5 = 405$$

b. 7 divisions.

$$g_8 = 5 \cdot 3^{8-1}$$

$$g_8 = 5 \cdot 3^7$$

$$g_8 = 5 \cdot 2187$$

$$g_8 = 10,935$$

~~c. 13 divisions.~~

~~d. 16 divisions.~~

Let's Practice!