

4.2

The Password Is... Operations!

Arithmetic and Geometric Sequences

LEARNING GOALS

In this lesson, you will:

- Determine the next term in a sequence.
- Recognize arithmetic sequences.
- Determine the common difference.
- Recognize geometric sequences.
- Determine the common ratio.

KEY TERMS

- arithmetic sequence
- common difference
- geometric sequence
- common ratio

1. Figure out the pattern for each sequence.
2. Then, calculate the next numbers in the sequence.
3. Record your results on pages 225 and 227 of your textbook.
4. Be prepared to share your results.

You have 3 minutes and 2 seconds!



A45, 90, 180, 360, 720, 1440,2880, ...

multiply by 2

B-4, -2, 0, 2, 4, 6,8, ...

add 2

C-2, -6, -18, -54, -162, -486,-1458, ...

multiply by 3

D2, 5, 10, 17, 26, 37,50, ...

add 3, then 5, then 7, etc.

E

$$4, \frac{7}{4}, -\frac{1}{2}, -\frac{11}{4}, \underline{-5}, \underline{-\frac{29}{4}},$$
$$\underline{-\frac{19}{2}}, \dots$$

subtract $\frac{9}{4}$

Find a common denominator, then compare the fractions

F

$$1234, 123.4, 12.34, 1.234, \underline{0.1234},$$
$$\underline{0.01234}, \underline{0.001234}, \dots$$

multiply by 0.1 or divide by 10

G

$$1, -2, 3, -4, 5, \underline{-6}, \underline{7},$$
$$\underline{-8}, \underline{9}, \dots$$

consecutive #s, but every other one is negative.

H

$$-20, -16, -12, -8, -4, \underline{0},$$
$$\underline{4}, \underline{8}, \dots$$

add 4

I

1, 10, 100, 1000, 10,000, 100,000, ...

multiply by 10

J

-5 , $-\frac{5}{2}$, $-\frac{5}{4}$, $-\frac{5}{8}$, $-\frac{5}{16}$, $-\frac{5}{32}$, ...

multiply by $\frac{1}{2}$

K

6.5, 5, 3.5, 2, 0.5, -1,

-2.5, ...

subtract 1.5

L

86, 85, 83, 80, 76, 71, 65, ...

subtract 1, then 2, then 3, ...

M

$-16, 4, -1, \frac{1}{4}, \underline{-\frac{1}{16}}, \underline{\frac{1}{64}}, \dots$

divide by -4

N

$1473.2, 1452.7, 1432.2, 1411.7, \underline{1391.2},$

$\underline{1370.7}, \underline{1350.2}, \dots$

subtract 20.5

O

$\sqrt{5}, 2, \sqrt{3}, \sqrt{2}, 1, 0, \sqrt{-1}, \underline{\sqrt{-2}},$

$\underline{\sqrt{-3}}, \dots$

square roots of consecutive integers

P

$-4, 12, -36, 108, \underline{-324}, \underline{972}, \dots$

multiply by -3

PROBLEM 2 Arithmetic, My Dear Watson!



You can describe a pattern as adding a constant to, or subtracting a constant from each term to determine the next term for some sequences. For other sequences, you can describe the pattern as multiplying or dividing each term by a constant to determine the next term. Still other sequences cannot be described either way.

6, 9, 12, 15, ...
Add 3 so the
common difference
or $d = 3$

15, 11, 7, 3, ...
Subtract 4 so the
common difference
 $d = -4$

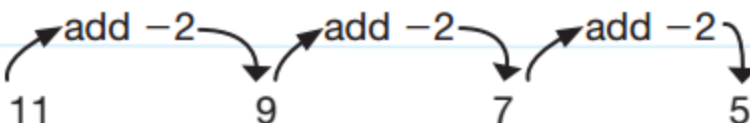
An **arithmetic sequence** is a sequence of numbers in which the difference between any two consecutive terms is a constant. In other words, it is a sequence of numbers in which a positive or negative constant is added to each term to produce the next term. This positive or negative constant is called the **common difference**. The common difference is typically represented by the variable d .

Consider the sequence shown.

11, 9, 7, 5, ...

The pattern is to add the same negative number, -2 , to each term to determine the next term.

Sequence: 11 , 9 , 7 , 5 , ...



The diagram shows the sequence 11, 9, 7, 5, ... with arrows pointing from each term to the next. Each arrow is labeled 'add -2', illustrating the constant difference between terms.

This sequence is arithmetic and the common difference d is -2 .

If the number you ADD to each number in the sequence is the SAME or CONSTANT, then it's arithmetic.



1. Suppose a sequence has the same starting number as the sequence in the worked example, but its common difference is 4.

a. How would the pattern change?

Start with 11 and increase the sequence by 4 instead of decreasing it by 2.

b. Is the sequence still arithmetic? Why or why not?

Yes, because you keep adding 4 each time. The common difference stays the SAME.



c. If possible, write the first 5 terms of the new sequence.

11, 15, 19, 23, 27



2. Analyze the sequences you cut out in Problem 1, *What Comes Next, and How Do You Know?* Look at the sequences on pages 225 and 227.
- a. List those sequences that are arithmetic.

B, E, H, K, N

Now let's look at each of those and find the common difference.

B

$-4, -2, 0, 2, \underline{4}, \underline{6},$
 $\underline{8}, \dots$

add 2

Think about it!

$$-2 - (-4) = -2 + 4 = 2$$

$$0 - (-2) = 0 + 2 = 2$$

Arithmetic: $d = 2$

E

$4, \frac{7}{4}, -\frac{1}{2}, -\frac{11}{4}, \underline{-5}, \underline{-\frac{29}{4}},$
 $\underline{-\frac{19}{2}}, \dots$

subtract $\frac{9}{4}$

Think about it!

$$\frac{7}{4} - 4 = \frac{7}{4} - \frac{16}{4} = -\frac{9}{4}$$

$$-\frac{1}{2} - \frac{7}{4} = -\frac{2}{4} - \frac{7}{4} = -\frac{9}{4}$$

Arithmetic: $d = -\frac{9}{4}$

H

-20, -16, -12, -8, -4, 0,

4, 8, ...

add 4

Arithmetic: $d = 4$

K

6.5, 5, 3.5, 2, 0.5, -1,

-2.5, ...

subtract 1.5

Arithmetic: $d = -1.5$

N

1473.2, 1452.7, 1432.2, 1411.7, 1391.2,

1370.7, 1350.2, ...

subtract 20.5

Arithmetic: $d = -20.5$