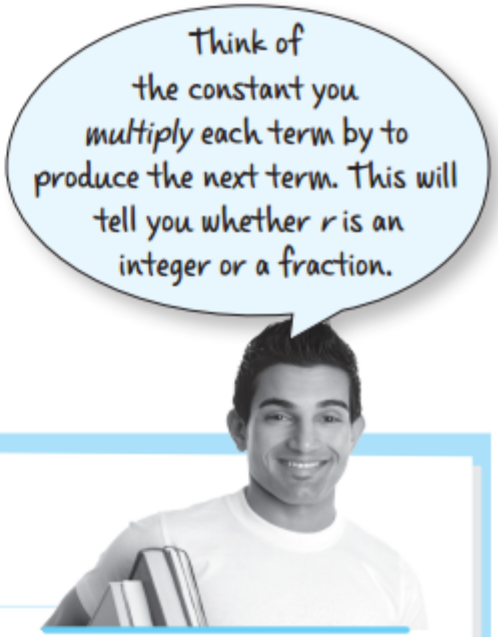
A **geometric sequence** is a sequence of numbers in which the ratio between any two consecutive terms is a constant. In other words, it is a sequence of numbers in which you multiply each term by a constant to determine the next term. This integer or fraction constant is called the **common ratio**. The common ratio is represented by the variable *r*.



Consider the sequence shown.

1, 2, 4, 8, . . .

The pattern is to multiply each term by the same number, 2, to determine the next term.

Sequence: 1 (multiply by 2) 2 (multiply by 2) 4 (multiply by 2) 8, . . .

This sequence is geometric and the common ratio *r* is 2.

If the number you multiply by is the SAME or CONSTANT, then the sequence is GEOMETRIC.

To find *r*, divide a number in the sequence by the previous number. $\frac{2}{1} = 2$ and $\frac{4}{2} = 2$, so $r = 2$.

3. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is 3.

What happens when *r* changes from 2 to 3? Do the terms stay the same?

a. How would the pattern change?

The terms in the sequence will be different. They will increase more rapidly.

b. Is the sequence still geometric? Explain your reasoning.

Yes. The common ratio remains constant.

c. If possible, write the first 5 terms for the new sequence.

1, 3, 9, 27, 81

4. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is $\frac{1}{3}$.

What happens to the sequence when r is $\frac{1}{3}$? Do the terms stay the same?

a. How would the pattern change?

The terms will be different. The sequence will decrease.

b. Is the sequence still geometric? Why or why not?

Yes. The common ratio remains constant.

c. If possible, write the first 6 terms for the new sequence.

$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \frac{1}{243}$

5. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is -2. What happens when r is -2? Do the terms stay the same?

a. How would the pattern change?

The sequence alternatively increases and decreases because the sign changes with every other number.

b. Is the sequence still geometric? Explain your reasoning.

Yes. The common ratio remains constant.

c. If possible, write the first 6 terms for the new sequence.

1, −2, 4, −8, 16, −32

Skip Problem 6.

7. Analyze the sequences you cut out in Problem 1, *What Comes Next, and How Do You Know*? again. Look at the sequences on pages 225 and 227.

a. List those sequences that are geometric.

A, C, F, I, J, M, P.

Now let's look at each of those and find the common ratio.

A

45, 90, 180, 360, 720, 1440,

2880, . . .

multiply by 2

geometric: $r = 2$

Think about it!

$\frac{90}{45} = 2$

C

−2, −6, −18, −54, −162, −486,

−1458, . . .

multiply by 3

geometric: $r = 3$

Think about it!

$\frac{-6}{-2} = 3$

F

1234, 123.4, 12.34, 1.234, 0.1234,

0.01234, 0.001234, . . .

multiply by 0.1

geometric: $r = 0.1$

Think about it!

$\frac{123.4}{1234} = 0.1$

I

1, 10, 100, 1000, 10,000, 100,000, . . .

multiply by 10

geometric: $r = 10$

Think about it!

$\frac{10}{1} = 10$

J

$-5, -\frac{5}{2}, -\frac{5}{4}, -\frac{5}{8}, \underline{-\frac{5}{16}}, \underline{-\frac{5}{32}}, \ldots$

multiply by $\frac{1}{2}$

$-\frac{5}{2} \div -\frac{5}{1}$

geometric: $r = \frac{1}{2}$

$-\frac{5}{2} \times -\frac{1}{5} = \frac{1}{2}$

M

$-16, 4, -1, \frac{1}{4}, \underline{-\frac{1}{16}}, \underline{\frac{1}{64}}, \ldots$

divide by -4

geometric: $r = -\frac{1}{4}$

$\frac{4}{-16} = -\frac{1}{4}$

P

$-4, 12, -36, 108, \underline{-324}, \underline{972}, \ldots$

multiply by -3

geometric: $r = -3$

$\frac{12}{-4} = -3$

8. Consider the sequences from Problem 1 that are neither arithmetic nor geometric.

a. List these sequences. Look at the sequences on pages 225 and 227.

D, G, L, O

b. Explain why these sequences are neither arithmetic nor geometric.

These sequences are neither arithmetic nor geometric because there is no common difference or common ratio for any of these sequences.