2.4

We're Shipping Out!

Solving and Graphing Compound Inequalities

LEARNING GOALS

In this lesson, you will:

- Write simple and compound inequalities.
- Graph compound inequalities.
- Solve compound inequalities.

KEY TERMS

- compound inequality
- solution of a compound inequality
- conjunction
- disjunction

PROBLEM 1 GoodSportsBuys.com

GoodSportsBuys.com is an online store that offers discounts on sports equipment to high school athletes. When customers buy items from the site, they must pay the cost of the items as well as a shipping fee. At GoodSportsBuys.com, a shipping fee is added to each order based on the total cost of all the items purchased. This table provides the shipping fee categories for GoodSportsBuys.com.

Total Cost of Items	Shipping Fee
$0.01 up to and including $20	$6.50
More than $20 up to and including $50	$9.00
Between $50 and $75	$11.00
From $75 up to, but not including, $100	$12.25
$100 or more	$13.10

1. What is the least amount a customer can spend on items and pay $6.50 for shipping?
   $0.01
2. What is the greatest amount a customer can spend on items and pay $6.50 for shipping?
   $20.00
3. What is the shipping fee if Sarah spends exactly $75.00 on items? Explain your reasoning.
   $12.25
4. Harvey says he will spend $13.10 on shipping fees if he spends exactly $100 on items. Is he correct? Explain your reasoning. Yes

5. Consider the table of shipping costs to complete each statement using the phrase "greater than," "less than," "greater than or equal to," or "less than or equal to."

a. You will pay $6.50 in shipping fees if you spend:

Total cost ≥ $0.01 AND total cost ≤ $20

b. You will pay $9.00 in shipping fees if you spend:

Total cost > $20 AND total cost ≤ $50

c. You will pay $11.00 in shipping fees if you spend:

Total cost > $50 AND total cost < $75

d. You will pay $12.25 in shipping fees if you spend:

Total cost ≥ $75 AND total cost < $100

e. You will pay $13.10 in shipping fees if you spend:

Total cost ≥ $100

A **compound inequality** is an inequality that is formed by the union, "or," or the intersection, "and," of two simple inequalities.

6. You can use inequalities to represent the various shipping fee categories at GoodSportsBuys.com. If you let x represent the total cost of items purchased, you can write an inequality to represent each shipping fee category. Complete each inequality using an inequality symbol.

a. \$6.50 shipping fees: $x \geq$ \$0.01 and $x \leq$ \$20 — $\$0.01 \leq x \leq \$20$

b. \$9.00 shipping fees: $x >$ \$20 and $x \leq$ \$50 — $\$20 < x \leq \$50$

c. \$11.00 shipping fees: $x >$ \$50 and $x <$ \$75 — $\$50 < x < \$75$

d. \$12.25 shipping fees: $x \geq$ \$75 and $x <$ \$100 — $\$75 \leq x < \$100$

e. \$13.10 shipping fees: $x \geq$ \$100

7. Identify the inequalities in Question 6 that are compound inequalities.

a, b, c, and d

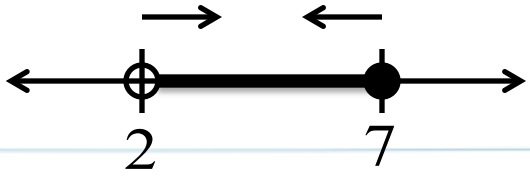
Compact form of "AND" inequalities

It's easy! Double flip the first term & combine the two equations.

Let's consider two examples of compound inequalities.

Let's Graph These!!!

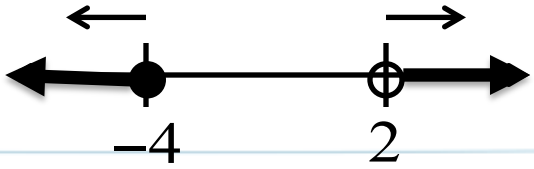
$x > 2$ and $x \leq 7$



This inequality is read as "all numbers greater than 2 and less than or equal to 7." This inequality can also be written in the compact form of $2 < x \leq 7$.

"AND" means between the two endpoints.

$x \leq -4$ or $x > 2$



This inequality is read as "all numbers less than or equal to -4 *or* greater than 2."

"OR" means it's one or the other solution.

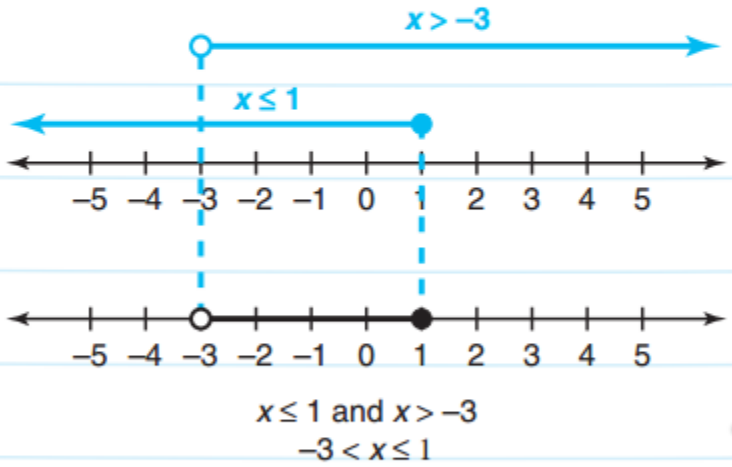
Skip to Page 117

The compound inequality shown involves "and" and is a conjunction.

$x \leq 1$ and $x > -3$

Represent each part above the number line.

"AND" inequalities have solutions that overlap!



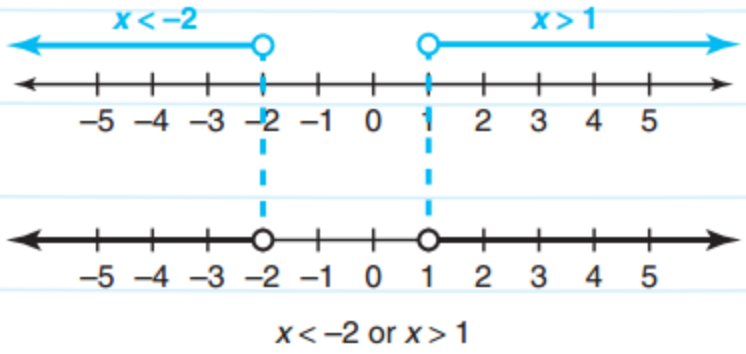
Does this make sense??? ☺

The solution is the region that satisfies both inequalities. Graphically, the solution is the overlapping, or the intersection, of the separate inequalities.

The compound inequality shown involves "or" and is a disjunction.

$x < -2$ or $x > 1$

Represent each part above the number line.



"OR" inequalities have solutions that don't overlap!

Does this make sense??? ☺

The solution is the region that satisfies either inequality. Graphically, the solution is the union, or all the regions, of the separate inequalities.

2. Consider the two worked examples in a different way.

a. If the compound inequality in the first worked example was changed to the disjunction, $x \leq 1$ or $x > -3$, how would the solution set change? Explain your reasoning.

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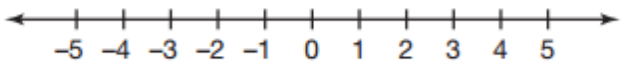

b. If the compound inequality in the second worked example was changed to the conjunction, $x < -2$ and $x > 1$, how would the solution set change? Explain your reasoning.

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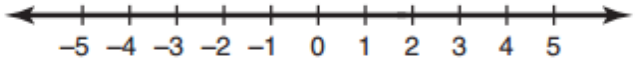
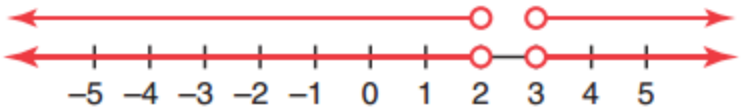


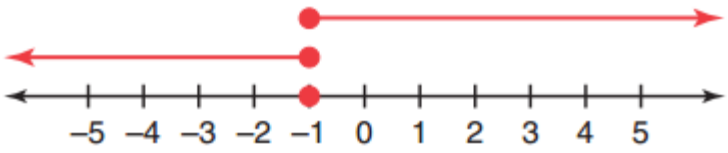
3. Represent the solution to each compound inequality on the number line shown. Then, write the final solution that represents the graph.

a. $x < 2$ or $x > 3$



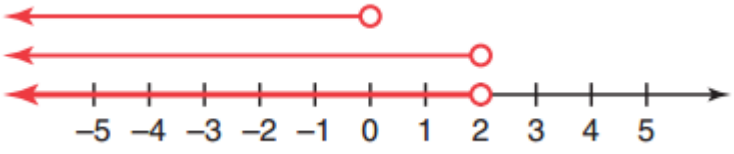
x < 2 or x > 3 is your solution.

b. $-1 \geq x \geq -1$



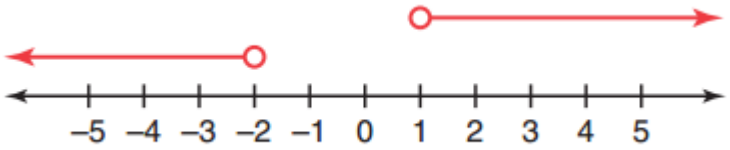
$x = -1$

c. $x < 0$ or $x < 2$



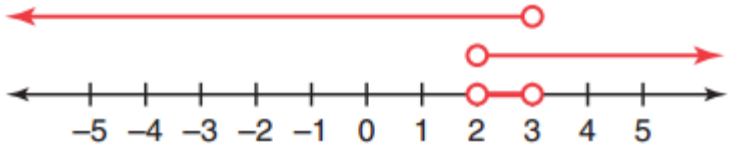
$x < 2$

d. $x > 1$ and $x < -2$



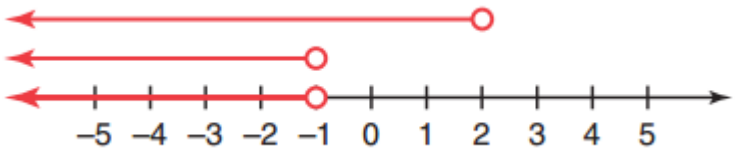
There is NO solution because the graphs don't overlap.

e. $x < 3$ and $x > 2$



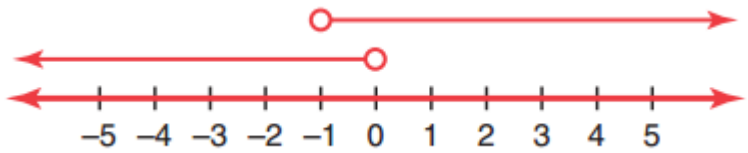
$2 < x < 3$

f. $x < 2$ and $x < -1$



$x < -1$

g. $x > -1$ or $x < 0$



All real numbers because you need to include both solutions.

To solve a compound inequality written in compact form, isolate the variable between the two inequality signs, and then graph the resulting statement. To solve an inequality involving "or," simply solve each inequality separately, keeping the word "or" between them, and then graph the resulting statements.

4. Solve and graph each compound inequality showing the steps you performed. Then, write the final solution that represents the graph.

a. $6 < x - 6 \leq 9$

$+6 \quad +6 \quad +6$

$12 < x \leq 15$

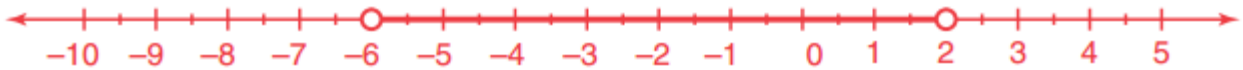
Repeat each step 3 times.

b. $-2 < -x < 6$

Divide by a negative #, flip both inequality signs.

$2 > x > -6 \rightarrow -6 < x < 2$

Double flip! Order #s from least to greatest.



~~**c.** $-4 \leq -3x + 1 \leq 12$~~

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f. $1 + 6x > 11$ or $x - 4 < -5$

$$\begin{array}{r} 1+6x>11 \\ -1 \quad -1 \\ \hline 6x>10 \end{array} \qquad \textit{or} \qquad \begin{array}{r} x-4<-5 \\ +4 \quad +4 \\ \hline x<-1 \end{array}$$

Solve and graph each inequality separately.

$$x > \frac{10}{6}$$

$$x > \frac{5}{3}$$