2.3

Scouting for Prizes!
Modeling Linear Inequalities

What is an inequality?
<, >, ≤, or ≥

LEARNING GOALS

In this lesson, you will:

- Write and solve inequalities.
- Analyze a graph on a coordinate plane to solve problems involving inequalities.
- Interpret how a negative rate affects how to solve an inequality.

KEY TERM

- solve an inequality

PROBLEM 1   Popcorn Pays Off

Alan's camping troop is selling popcorn to earn money for an upcoming camping trip. Each camper starts with a credit of $25 toward his sales, and each box of popcorn sells for $3.75.

Alan can also earn bonus prizes depending on how much popcorn he sells. The table shows the different prizes for each of the different sales levels. Each troop member can choose any one of the prizes at or below the sales level.

Sales (dollars)	Gift Cards (2 of each value)	Bonus Prizes
$250	$10	
$350	$15	
$450	$20	
$600		Cyclone Sprayer
$650	$30	
$850	$40	
$1100	$55	
$1300	$75	
$1500		Choose your prize!
$1800	$110	
$2300	$150	
$2500		6% toward college scholarship

What Do We Know?

Starting Point = $25

Rate of Change = $3.75/box

of popcorn sold

1. Write a function, $f(b)$, to show Alan's total sales as a function of the number of boxes of popcorn he sells.

$f(b) = 3.75b + 25$ ← Slope-intercept Form
$y = mx + b$

2. Analyze the function you wrote.

    a. Identify the independent and dependent quantities and their units.

    Independent Quantity (or b) = # of popcorn boxes sold
    Dependent Quantity (or f(b)) = total sales (in dollars)

    b. What is the rate of change and what does it represent in this problem situation?

    Rate of Change = $3.75.   It represents the cost of each box of popcorn.

    c. What is the y-intercept and what does it represent in this problem situation?

    Starting Point or y-intercept = $25.
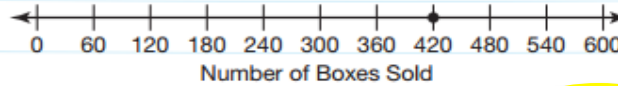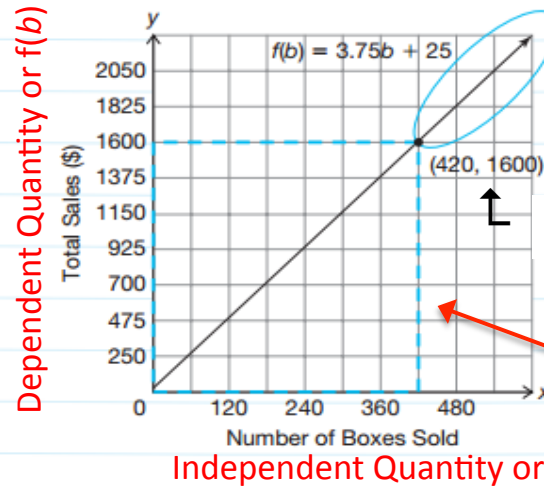   Every troop member starts with a $25 credit toward the total sales.

Now, let's analyze your function represented on a graph.

Let's Solve by Graphing

1) Draw a horizontal line at f(b) = 1600.

2) Find the POI (420, 1600).

3) Draw a vertical line from the POI to the x-axis to find the solution, b = 420.

The graph shown represents the change in the total sales as a function of boxes sold. The oval and box represent the total sales at specific intervals.


$f(b) = 3.75b + 25$

Dependent Quantity or f(b)
Total Sales ($)

2050
1825
1600
1375
1150
925
700
475
250
0

(420, 1600)

120   240   360   480

Number of Boxes Sold
Independent Quantity or b

The oval represents # of boxes sold > 420 or total sales > 1600

Point–of–Intersection (POI)

The box represents # of boxes sold ≤ 420 or total sales ≤ 1600

Number line:
0 60 120 180 240 300 360 420 480 540 600
Number of Boxes Sold

The point at (420, 1600) means that at 420 boxes sold, the total sales is equal to $1600. This is represented on the number line as a closed point at 420. When f(b) = 1600, then b = 420.

The box represents all the numbers of boxes sold, b, that would earn Alan $1600 or less. When f(b) ≤ 1600 then b ≤ 420.

Number line (boxed):
0 60 120 180 240 300 360 420 480 540 600
Number of Boxes Sold
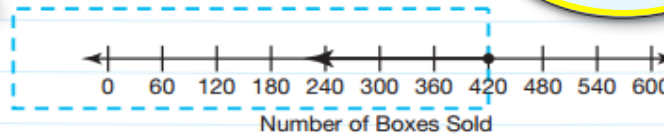
The oval represents all the numbers of boxes sold, b, that would earn Alan more than $1600. When f(b) > 1600, then b > 420.
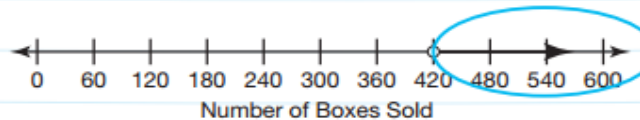
Number line (ovaled):
0 60 120 180 240 300 360 420 480 540 600
Number of Boxes Sold

3. Explain the difference between the open and closed circles on the number lines.

O **or Open Circle** = the point IS NOT included in the solution
● **or Closed Circle** = the point IS included in the solution

4. Use the graph to answer each question. Write an equation or inequality statement for each.

a. How many boxes would Alan have to sell to earn at least $925?

$b \geq 240$
Alan would have to sell **at least** 240 boxes

b. How many boxes would Alan have to sell to earn less than $2050?

$b < 540$
Alan would have to sell **fewer than** 540 boxes.

c. How many boxes would Alan have to sell to earn exactly $700?

$b = 180$
Alan would have to sell **exactly** 180 boxes.

How does determining the intersection point help you determine your answers?