## What Goes Up Must Come Down Analyzing Linear Functions

## LEARNING GOALS

In this lesson, you will:

- Complete tables and graphs, and write equations to model linear situations.

There are 3 ways to

- Analyze multiple representations of linear relationships.
- Identify units of measure associated with linear relationships.
- Determine solutions to linear functions using intersection points and properties of equality.
- Determine solutions using tables, graphs, and functions.
- Compare and contrast different problem-solving methods.
- Estimate solutions to linear functions.
- Use a graphing calculator to analyze functions and their graphs.


## PROBLEM 1 As We Make Our Final Descent



At 36,000 feet, the crew aboard the 747 airplane begins making preparations to land.
The plane descends at a rate of 1500 feet per minute until it lands.

1. Compare this problem situation to the problem situation in Lesson 2.1, The Plane!

How are the situations the same? How are they different?
Same: $\mathrm{IQ}=$ Time \& DQ = Height.
Differences: ROC is negative \& the starting point is not zero.
2. Complete the table to represent this problem situation.

| Quantity | Independent Quantity |
| :--- | :---: |
|  | Time |
|  | Dependent Quantity |
|  | minutes |
| 0 | Height |
| 2 | 36,000 |
| 4 | 33,000 |

3. Write a function, $g(t)$, to represent this problem situation.

$$
\begin{gathered}
g(t)=36000-1500 t \\
\text { or } \\
g(t)=-1500 t+36000 \\
\uparrow \\
\begin{array}{c}
\text { Slope-intercept } \\
\text { Form }
\end{array}
\end{gathered}
$$

The plane is starting at 36,000 feet.

The rate of change is
-1500 feet per minute.

We are losing altitude!
4. Complete the table shown. First, determine the unit of measure for each expression. Then, describe the contextual meaning of each part of the function. Finally, choose a term from the word box to describe the mathematical meaning of each part of the function.

| input value |  |  |  |
| :--- | :--- | :--- | :--- |
|  | $y$-intercept |  | rate of change |
|  |  | $x$-intercept |  |


|  | Description |  |  |
| :---: | :---: | :---: | :---: |
| Expression | Units | Contextual <br> Meaning | Mathematical <br> Meaning |
| $t$ | minutes | Amt of time the <br> plane descends | input value |
| -1500 | $\frac{\text { feet }}{\text { minute }}$ | \# of feet the plane <br> descends per min | rate of change |
| $-1500 t$ | feet | \# of feet the plane <br> descended |  |
| 36,000 | feet | Plane's initial <br> height | y-intercept |
| $-1500 t+36,000$ | feet | Height of the <br> plane | output value |

5. Graph $g(t)$ on the coordinate plane shown.
output

input

You have just represented the As We Make Our Final Descent scenario in different ways:

- numerically, by completing a table,
- algebraically, by writing a function, and
- graphically, by plotting points.

Let's consider how to use each of these representations to answer questions about the problem situation.

6. Determine how long will it take the plane to descend to 14,000 feet.
a. Use the table to determine how long it will take the plane to descend to 14,000 feet.
(Look at the table on page 88.) It takes 12-20 minutes.
b. Graph and label $y=14,000$ on the coordinate plane. Then determine the intersection point. Explain what the intersection point means in terms of this problem situation.
Use the graph on page 89. Draw a horizontal line where $y=14,000$.
The point-of-intersection (POI) is $12-16$ minutes, or approximately 14 minutes.
c. Substitute 14,000 for $\mathrm{g}(\mathrm{t})$ and solve the equation for t . Interpret your solution in terms of this problem
situation.

$$
\begin{aligned}
14000 & =-1500 t+36,000 \\
-22,000 & =-1500 t \\
14 . \overline{6} & =t
\end{aligned}
$$

d. Compare and contrast your solutions using the table, graph, and the function. What do you notice? Explain your reasoning.
The table produces an estimate.
The graph gives you an approximation.
The function/equation results in an exact solution.
7. Determine how long it will take the plane to descend to 24,000 feet.
a. Use the table to determine how long it will take the plane to descend to 24,000 feet. (Look at the table on page 88.) It takes 6-12 minutes.
b. Graph and label $y=24,000$ on the coordinate plane. Then determine the intersection point. Explain what the intersection point means in terms of this situation.
Use the graph on page 89. Draw a horizontal line where $y=24,000$.
The POI is $(8,24000)$. The plane descends to 24,000 feet in 8 minutes.
c. Substitute 24,000 for $g(t)$ and solve the equation for $t$. Interpret your solution in terms of this situation.

$$
\begin{aligned}
24000 & =-1500 t+36,000 \\
-12,000 & =-1500 t \\
8 & =t
\end{aligned}
$$

c. Compare and contrast your solutions using the table, graph, and the function. What do you notice? Explain your reasoning.
Table $=$ an estimated time
Graph $=$ an exact time because the line intersects the corner of the coordinate grid when $t=8$
Function = exact time
8. For how many heights can you calculate the exact time using the:
a. table?

6 rows in the table $=$ you find the exact time for 6 different heights
b. graph?

4 places where the graph intersects a corner of the grid = you can find the exact time for 4 different heights
c. function?

You can always calculate an exact time for any given height.
9. Use the word bank to complete each sentence.


If I am given a dependent value and need to calculate an independent value of a linear function,
a. I can always use a table to determine an approximate value.
b. I can sometimes use a table to calculate an exact value.
c. I can_ulways use a graph to determine an approximate value.
d. I can $\qquad$ use a graph to calculate an exact value.
e. I can $\qquad$ use a function to determine an approximate value.
f. I can $\qquad$ use a function to calculate an exact value.

