PROBLEM 2 Analyzing Equations and Graphs

1. Complete the table shown for the problem situation described in Problem 1, *Analyzing Tables*. First, determine the unit of measure for each expression. Then, describe the contextual meaning of each part of the function. Finally, choose a term from the word box to describe the mathematical meaning of each part of the function.

output value	input value	rate of change

		What It Means	
Expression	Unit	Contextual Meaning	Mathematical Meaning
t	minutes	Amount of time the plane ascends	input value
1800	$\frac{\text{feet}}{\text{minute}}$	# of feet the plane climbs each minute	rate of change
$1800t$	feet	Height of the plane	output value

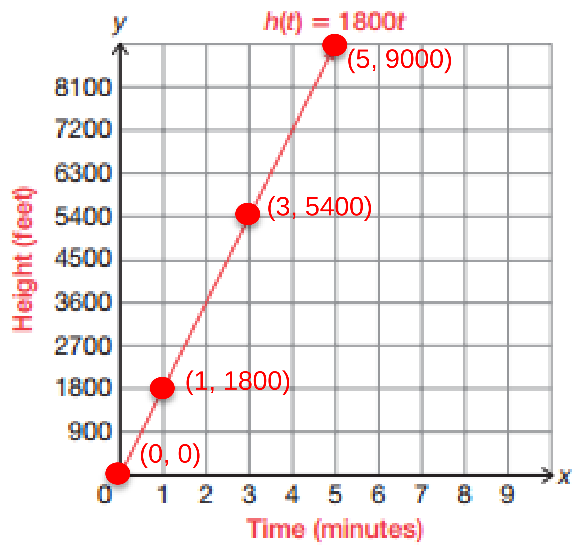
2. Write a function, $h(t)$, to describe the plane's height over time, t.

$H(t) = 1800t$

3. Which function family does $h(t)$ belong to? Is this what you predicted back in Problem 1, Question 3?

Function family = Linear
The graph is an increasing, continuous line.

4. Use your table and function to create a graph to represent the change in the plane's height as a function of time. Be sure to label your axes with the correct units of measure and write the function.



a. What is the slope of this graph? Explain how you know.

Slope = 1800/1. The plane climbs 1800 feet every minute.

b. What is the *x*-intercept of this graph? What is the *y*-intercept? Explain how you determined each intercept.

The x- and y-intercepts are both 0.

The intercept is where the line crosses the x- or y- axes.

c. What do the *x*- and *y*-intercepts mean in terms of this problem situation?

x-intercept = 0 minutes, y-intercept = 0 feet

The plane is on the ground waiting to takeoff.

Let's consider how to determine the height of the plane, given a time in minutes, using function notation.

To determine the height of the plane at 2 minutes using your function, substitute 2 for t every time you see it. Then, simplify the function.

$$h(t) = 1800t$$

Substitute 2 for t. $\longrightarrow$ $h(2) = 1800(2)$

$$h(2) = 3600$$

Two minutes after takeoff, the plane is at 3600 feet.

5. List the different ways the height of the plane is represented in the example.

6. Use your function to determine the height of the plane at each given time in minutes. Write a complete sentence to interpret your solution in terms of the problem situation.

a. $h(3) =$ 5400 ft

$h(3) = 1800(3)$
$h(3) = 5400$

b. $h(3.75) =$ 6750 ft

$h(3.75) = 1800(3.75)$

c. $h(5.1) =$ 9180 ft

$h(5.1) = 1800(5.1)$

d. $h(-4) =$ -7200 ft

$h(-4) = 1800(-4)$

Does this make sense? Can you have -4 minutes?

PROBLEM 3 Connecting Approaches

Now let's consider how to determine the number of minutes the plane has been flying (the input value) given a height in feet (the output value) using function notation.

To determine the number of minutes it takes the plane to reach 7200 feet using your function, substitute 7200 for $h(t)$ and solve.

$$h(t) = 1800t$$

Substitute 7200 for $h(t)$. →

$$7200 = 1800t$$

$$\frac{7200}{1800} = \frac{1800t}{1800}$$

$$4 = t$$

After takeoff, it takes the plane 4 minutes to reach a height of 7200 feet.

1. Why can you substitute 7200 for $h(t)$?

We know the plane reaches 7200 feet and $h(t)$ is the height of the plane.

2. Use your function to determine the time it will take the plane to reach each given height in feet. Write a complete sentence to interpret your solution in terms of the problem situation.

a. 5400 feet

$$5400 = 1800t$$

$$\frac{5400}{1800} = \frac{1800t}{1800}$$

$$3 = t$$

b. 9000 feet

$$9000 = 1800t$$

$$\frac{9000}{1800} = \frac{1800t}{1800}$$

$$5 = t$$

The plane will be at 9000 feet 5 minutes after takeoff.

c. 3150 feet

$3150 = 1800t$

$$\frac{3150}{1800} = \frac{1800t}{1800}$$

$1.75 = t$

d. 4500 feet

$4500 = 1800t$

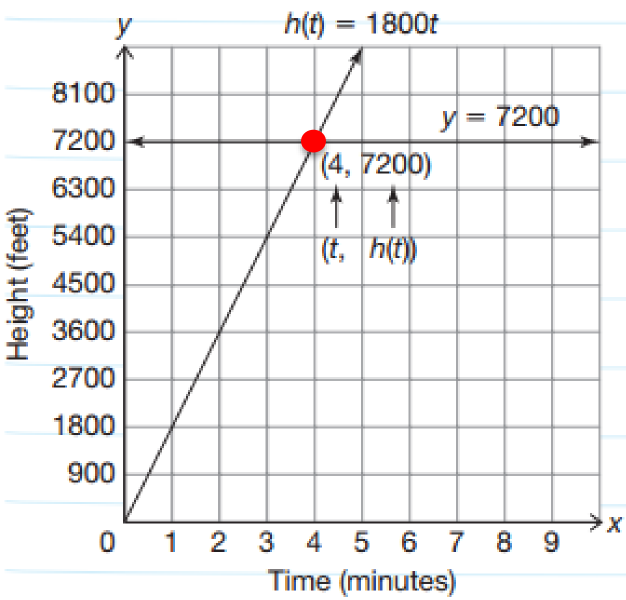
$$\frac{4500}{1800} = \frac{1800t}{1800}$$

$2.5 = t$

You can also use the graph to determine the number of minutes the plane is flying at a given height. Let y = given height of the plane and graph the horizontal line. The point where the horizontal line intersects the graph of the function is the solution.

To determine how many minutes it takes for the plane to reach 7200 feet using your graph, you need to determine the intersection of the two graphs represented by the equation $7200 = 1800t$.

First, graph each side of the equation and then determine the intersection point of the two graphs.



$$h(t) = 1800t$$

$$\begin{array}{ccc} 7200 & = & 1800t \\ \downarrow & & \downarrow \\ y = 7200 & & y = 1800x \end{array}$$

Solution: (4, 7200)

After takeoff, it takes the plane 4 minutes to reach a height of 7200 feet.

time height

3. What does $(t, h(t))$ represent?

The point-of-intersection. The place where the graphs of both functions meet. It is the solution.

4. Explain the connection between the form of the function h(t) = 1800t and the equation y = 1800x in terms of the independent and dependent quantities.

$y = 1800x$
x is independent
y is dependent

$h(t) = 1800t$
t is independent
$h(t)$ is dependent

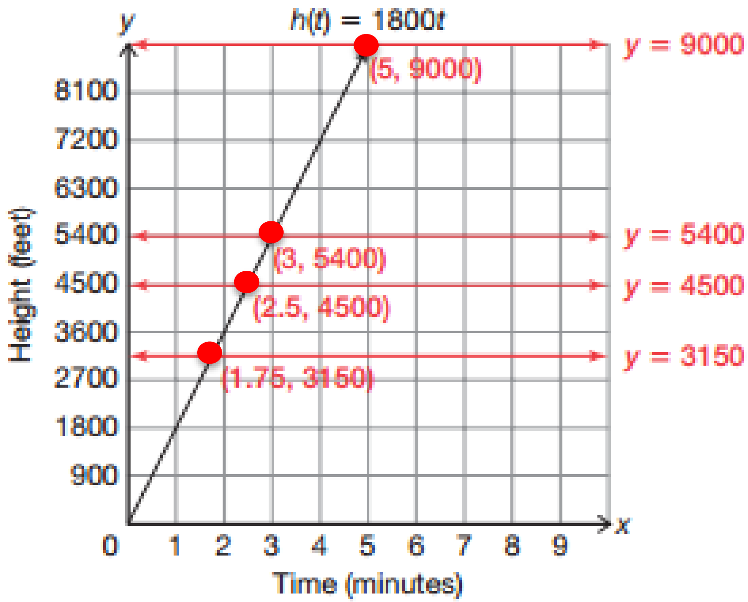
5. Use the graph to determine how many minutes it will take the plane to reach each height.

a. $h(t) = 5400$

b. $h(t) = 9000$

c. $h(t) = 3150$

d. $h(t) = 4500$



6. Compare and contrast your solutions using the graphing method to the solutions in Question 2, parts (a) through (d) where you used an algebraic method. What do you notice?

Algebraic method or equations = exact answer

Graphing method = an estimate or an exact answer