

Solving Quadratic Equations by Completing the Square

Main Ideas/Questions	Notes/Examples			
Perfect Square Trinomials	$x^2 + 6x + 9$	$x^2 - 10x + 25$	$x^2 + 12x + 36$	
WARM UP	What's the relationship between the middle and last term? How could you use the middle term to find the last term? Divide the middle term by 2. Then, square it to get the last term.			
	How could you turn $x^2 + 14x$ into a perfect square trinomial? $\left(\frac{14}{2}\right)^2$ Divide the middle term (b) by 2. Square it to find the last or constant term (c). $x^2 + 14x + 49$ is a perfect square trinomial.			
PRACTICE	Directions: Create a perfect square trinomial by finding the last term. Then, factor the trinomial.			
	1. $x^2 + 20x + \underline{100}$ $(x + 10)^2$	2. $x^2 - 4x + \underline{4}$ $(x - 2)^2$		
	$\left(\frac{20}{2}\right)^2 = (10)^2 = 100$	$\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$		
	3. $x^2 + 5x + \frac{25}{4}$ $\left(x + \frac{5}{2}\right)^2$ $\left(\frac{5}{2}\right)^2 = \frac{25}{4}$	4. $x^2 - 14x + \underline{49}$ $(x - 7)^2$ $\left(\frac{-14}{2}\right)^2 = (-7)^2 = 49$		

COMPLETING THE SQUARE

When you don't have a perfect square trinomial, you can create one!
This process is called **Completing the Square**.

Example

$$x^2 + 8x - 20 = 0$$

$$x^2 + 8x \underline{\quad} = 20$$

$$\left(\frac{8}{2}\right)^2 = (4)^2 = 16$$

$$x^2 + 8x + 16 = 20 + 16$$

$$(x+4)^2 = 36$$

$$\sqrt{(x+4)^2} = \pm\sqrt{36}$$

$$x+4 = \pm 6$$

$$x = -4 \pm 6$$

$$x = -4 + 6 = 2$$

$$x = -4 - 6 = -10$$

Steps

1. Rewrite as $x^2 + bx = c$ by moving the given c to the right side of the equation.
2. Find $\left(\frac{b}{2}\right)^2$. This will be the new c value.
3. Add $\left(\frac{b}{2}\right)^2$ to both sides of the equation to create a perfect square on the left side of the equation.
4. Factor the perfect square trinomial. Simplify the right side.
5. Take the square root of both sides of the equation.
6. Solve for x using the positive and negative square roots. These are the solutions.

YOU TRY!

Directions: Solve each quadratic equation by completing the square.

$$1. \quad x^2 - 6x - 16 = 0$$

$$x^2 - 6x + \underline{\quad} = 16$$

$$\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$$

$$x^2 - 6x + 9 = 16 + 9$$

$$(x-3)^2 = 25$$

$$\sqrt{(x-3)^2} = \pm\sqrt{25}$$

$$x-3 = \pm 5$$

$$x = 3 \pm 5$$

$$x = 3 + 5 \quad x = 3 - 5$$

$$x = 8 \quad x = -2$$

$$x = 8, x = -2$$

$$2. \quad x^2 - 2x - 5 = 0$$

$$x^2 - 2x + \underline{\quad} = 5$$

$$\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$$

$$x^2 - 2x + 1 = 5 + 1$$

$$(x-1)^2 = 6$$

$$\sqrt{(x-1)^2} = \pm\sqrt{6}$$

$$x-1 = \pm\sqrt{6}$$

$$x = 1 \pm\sqrt{6}$$

$$x = 1 \pm\sqrt{6}$$

$$3. x^2 + 71 = 18x - 9$$

$$x^2 - 18x + \underline{\quad} = -80$$

$$\left(\frac{18}{2}\right)^2 = 9^2 = 81$$

$$x^2 - 18x + 81 = -80 + 81$$

$$(x-9)^2 = 1$$

$$\sqrt{(x-9)^2} = \pm\sqrt{1}$$

$$x-9 = \pm 1$$

$$x = 9 \pm 1$$

$$x = 9 + 1 \quad x = 9 - 1$$

$$x = 10 \quad x = 8$$

$$x = 8, x = 10$$

$$4. x^2 + 8x + 11 = 0$$

$$x^2 + 8x + \underline{\quad} = -11$$

$$\left(\frac{8}{2}\right)^2 = 4^2 = 16$$

$$x^2 + 8x + 16 = -11 + 16$$

$$(x+4)^2 = 5$$

$$\sqrt{(x+4)^2} = \pm\sqrt{5}$$

$$x+4 = \pm\sqrt{5}$$

$$x = -4 \pm \sqrt{5}$$

$$x = -4 \pm \sqrt{5}$$

$$5. x^2 + 8x + 7 = 0$$

$$x^2 + 8x + \underline{\quad} = -7$$

$$\left(\frac{8}{2}\right)^2 = 4^2 = 16$$

$$x^2 + 8x + 16 = -7 + 16$$

$$(x+4)^2 = 9$$

$$\sqrt{(x+4)^2} = \pm\sqrt{9}$$

$$x+4 = \pm 3$$

$$x = -4 + 3 \quad x = -4 - 3$$

$$x = -1 \quad x = -7$$

$$x = -7, x = -1$$

$$6. x^2 - 6x + 4 = 0$$

$$x^2 - 6x + \underline{\quad} = -4$$

$$\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$$

$$x^2 - 6x + 9 = -4 + 9$$

$$(x-3)^2 = 5$$

$$\sqrt{(x-3)^2} = \pm\sqrt{5}$$

$$x-3 = \pm\sqrt{5}$$

$$x = 3 \pm \sqrt{5}$$

$$x = 3 \pm \sqrt{5}$$

Answers:

1. $x = 8, x = -2$

4. $x = -4 \pm \sqrt{5}$

2. $x = 1 \pm \sqrt{6}$

5. $x = -7, x = -1$

3. $x = 8, x = 10$

6. $x = 3 \pm \sqrt{5}$