Factored Form of a Quadratic Function

Let's Review

What is a quadratic function?

A polynomial of degree 2 (the highest exponent is 2). The graph is a "U" shaped curve called a parabola.

Examples: $5x^2 + 7$

 $6x^2 + 3x - 1$

 $9x^2$

What are 2 forms of writing a quadratic function?

Standard form	$y = ax^2 + bx + c$, where $a \neq 0$
Vertex form	$y = a(x - h)^2 + k$, where $a \ne 0$

3rd Form: Writing a Quadratic Function in Factored Form

 $y = a(x - r_1)(x - r_2)$, where $a \neq 0$

 r_1 and r_2 = x-coordinates of the solution, written as $(r_1, 0)$ and $(r_2, 0)$.

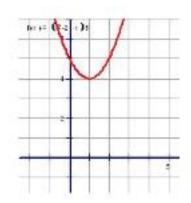
Solutions for Quadratic Functions

When you graph a quadratic equation, the solutions are the \underline{x} -intercepts or the point(s) where the parabola crosses the x-axis.

(3,0)

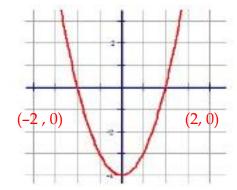
The x-intercepts also called the <u>zeros</u> or <u>roots</u>.

How many solutions does each parabola have?



No solution

1 solution



2 solutions

Can a parabola have more than two real solutions?

Why is there no solution?

There are no *x*-intercepts.

A quadratic equation can have $\underline{0}$, $\underline{1}$, or $\underline{2}$ real solutions.

Solving Quadratic Functions in Factored Form

Use the *Zero Product Property*: If ab = 0, then a = 0 or b = 0.

Think about It!

If 4gb = 0, what is the value of b? 0

Let's Look at an Example! How do we find a solution?

If
$$(x + 4)(x - 3) = 0$$
, then $(x + 4) = 0$ or $(x - 3) = 0$

$$x + 4 = 0$$

$$-4 - 4$$

$$x = -4$$

$$x - 3 = 0$$

$$+3 + 3$$

$$x = 3$$

Point out that this just a sign change, x + 4 = 0 means x = -4

Solutions: (-4,0) and (3,0)

Is the v-coordinate always 0? Why?

Find the solution(s) or x-intercept(s) for each quadratic function written in factored form.

1.
$$(x + 7)(3x - 1) = 0$$

$$x + 7 = 0$$
 $3x - 1 = 0$

$$x = -7 \qquad x = \frac{1}{3}$$

2.
$$(4s + 8)(s + 9) = 0$$

$$4s + 8 = 0$$
 $s + 9 = 0$ $j = 0$ $j - 8 = 0$

$$s = -2$$
 $j = 0$ $j = 8$

3.
$$j(j-8)=0$$

$$j = 0 \qquad j - 8 = 0$$

$$j = 0 \qquad \qquad j = 8$$

Solutions: (-7, 0) and

$$\left(\frac{1}{3},0\right)$$

4.
$$(x-4)(3x-12)=0$$

$$x - 4 = 0$$
 $3x - 12 = 0$

$$x = 4$$
 $x = 4$

Solution: (4, 0)

5.
$$\frac{1}{2}(x-4)(x+1)=0$$

$$x - 4 = 0 \qquad x + 1 = 0$$

$$x = 4$$
 $x = -1$

Solutions: (4, 0) and (-1, 0)

6.
$$-(x-3)(x-11) = 0$$

$$x - 3 = 0$$
 $x - 11 = 0$

$$x = 3$$
 $x = 11$

Solutions: (3, 0) and (11, 0)

Writing a Quadratic Function in Factored Form

We need to know two things!

1. Does the parabola open up or down?

2. What are the *x*-intercepts?

If the parabola opens DOWN, add a "-" in front of the factors.

Let's Look at an Example! How do we write a quadratic function in factored form?

The parabola opens UP and x-intercepts are (2, 0) and (4, 0).

If (2, 0) is a solution, then x = 2.

If
$$(4, 0)$$
 is a solution, then $x = 4$.

$$x = 2$$

$$\frac{-2}{x - 2} = \frac{-2}{0}$$

$$x = 4$$

$$\frac{-4}{x - 4} = \frac{-4}{0}$$

Work backwards! Point out that this just a sign change, x = 2means x - 2 = 0.

$$f(x) = \underline{(x-2)(x-4)}$$

Write a quadratic equation in factored using the given information.

1. The parabola opens DOWN and the *x*-intercepts are (–3, 0) and (1, 0).

$$f(x) = -(x+3)(x-1)$$

2. The parabola opens UP and the *x*-intercepts are (3.5, 0) and (-4.3, 0).

$$f(x) = (x - 3.5)(x + 4.3)$$

3. The parabola opens DOWN and the *x*-intercepts are (0, 0) and (5, 0).

$$f(x) = -x(x-5)$$
 or $f(x) = -(x-0)(x-5)$

4. The parabola opens UP and the *x*-intercepts are $\left(-\frac{1}{2},0\right)$ and $\left(-\frac{3}{4},0\right)$.

$$f(x) = \left(x + \frac{1}{2}\right)\left(x + \frac{3}{4}\right)$$

5. The parabola opens DOWN and the *x*-intercepts are (4, 0) and (–2, 0).

$$f(x) = -(x - 4)(x + 2)$$

6. The parabola opens UP and the *x*-intercepts are (1, 0) and $\left(\frac{2}{3}, 0\right)$.

$$f(x) = (x-1)\left(x - \frac{2}{3}\right)$$