Algebra 1: 11.4 Notes & CW/HW **Name** ______________________________ **Period** _____

Factored Form of a Quadratic Function

<u>Let's Review</u>

What is a quadratic function?

A polynomial of degree 2 (the highest exponent is 2). The graph is a "U" shaped curve called a parabola.

Examples: $5x^2 + 7$ $6x^2 + 3x - 1$ $9x^2$

What are 2 forms of writing a quadratic function?

Standard form	$y = ax^2 + bx + c$, where $a \neq 0$
Vertex form	$y = a(x - h)^2 + k$, where $a \neq 0$

<u>3rd Form: Writing a Quadratic Function in Factored Form</u>

$y = a(x - r_1)(x - r_2)$, where $a \neq 0$

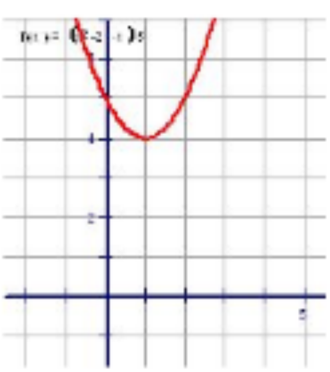
r_1 and r_2 = x-coordinates of the solution, written as $(r_1, 0)$ and $(r_2, 0)$.

<u>Solutions for Quadratic Functions</u>

When you graph a quadratic equation, the solutions are the <u>*x*-intercepts</u> or the point(s) where the parabola crosses the *x*-axis.

The *x*-intercepts also called the <u>zeros</u> or <u>roots.</u>

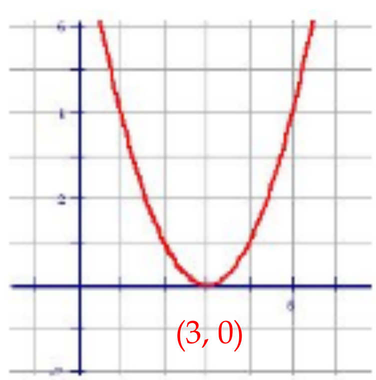
How many solutions does each parabola have?



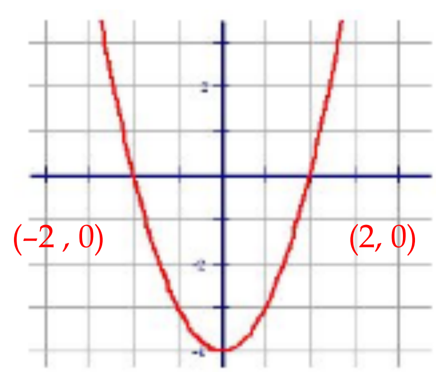
<u>No solution</u>

Why is there no solution?

There are no *x*-intercepts.



<u>1 solution</u>



<u>2 solutions</u>

Can a parabola have more than two real solutions?

A quadratic equation can have <u>0</u>, <u>1</u>, or <u>2</u> real solutions.

Solving Quadratic Functions in Factored Form

Use the ***Zero Product Property***: If $ab = 0$, then $a = 0$ or $b = 0$.

Think about It! If $4 \bullet b = 0$, what is the value of b? 0

Let's Look at an Example! How do we find a solution?

If $(x + 4)(x - 3) = 0$, then $(x + 4) = 0$ **or** $(x - 3) = 0$

$$\begin{array}{rr} x + 4 = 0 \\ \underline{-4} \quad \underline{-4} \\ x = -4 \end{array} \qquad \begin{array}{rr} x - 3 = 0 \\ \underline{+3} \quad \underline{+3} \\ x = 3 \end{array}$$

Solutions: (–4 , 0) and (3, 0)

Point out that this just a sign change, $x + 4 = 0$ means $x = -4$

Is the y-coordinate always 0? Why?

Find the solution(s) or x-intercept(s) for each quadratic function written in factored form.

1. $(x + 7)(3x - 1) = 0$

   $x + 7 = 0$ $\quad 3x - 1 = 0$

   $x = -7$ $\quad x = \frac{1}{3}$

   Solutions: (–7, 0) and $\left(\frac{1}{3}, 0\right)$

2. $(4s + 8)(s + 9) = 0$

   $4s + 8 = 0$ $\quad s + 9 = 0$

   $s = -2$ $\quad s = -9$

   Solutions: (–2, 0) and (–9, 0)

3. $j(j - 8) = 0$

   $j = 0$ $\quad j - 8 = 0$

   $j = 0$ $\quad j = 8$

   Solutions: (0, 0) and (8, 0)

4. $(x - 4)(3x - 12) = 0$

   $x - 4 = 0$ $\quad 3x - 12 = 0$

   $x = 4$ $\quad x = 4$

   Solution: (4, 0)

5. $\frac{1}{2}(x - 4)(x + 1) = 0$

   $x - 4 = 0$ $\quad x + 1 = 0$

   $x = 4$ $\quad x = -1$

   Solutions: (4, 0) and (–1, 0)

6. $-(x - 3)(x - 11) = 0$

   $x - 3 = 0$ $\quad x - 11 = 0$

   $x = 3$ $\quad x = 11$

   Solutions: (3, 0) and (11, 0)

<u>Writing a Quadratic Function in Factored Form</u>

We need to know two things!

If the parabola opens DOWN, add a "–" in front of the factors.

1. Does the parabola open up or down?
2. What are the x-intercepts?

Let's Look at an Example! How do we write a quadratic function in factored form?

The parabola opens UP and x-intercepts are (2, 0) and (4, 0).

If (2, 0) is a solution, then $x = 2$.

$$\begin{array}{rcr} x & = & 2 \\ \underline{-2} & & \underline{-2} \\ x-2 & = & 0 \end{array}$$

If (4, 0) is a solution, then $x = 4$.

$$\begin{array}{rcr} x & = & 4 \\ \underline{-4} & & \underline{-4} \\ x-4 & = & 0 \end{array}$$

Work backwards! Point out that this just a sign change, $x = 2$ means $x - 2 = 0$.

$f(x) = \underline{(x-2)(x-4)}$

Write a quadratic equation in factored using the given information.

1. The parabola opens DOWN and the x-intercepts are (–3, 0) and (1, 0).

   $f(x) = -(x+3)(x-1)$

2. The parabola opens UP and the x-intercepts are (3.5, 0) and (–4.3, 0).

   $f(x) = (x-3.5)(x+4.3)$

3. The parabola opens DOWN and the x-intercepts are (0, 0) and (5, 0).

   $f(x) = -x(x-5)$ or $f(x) = -(x-0)(x-5)$

4. The parabola opens UP and the x-intercepts are $\left(-\frac{1}{2},0\right)$ and $\left(-\frac{3}{4},0\right)$.

   $f(x)=\left(x+\frac{1}{2}\right)\left(x+\frac{3}{4}\right)$

5. The parabola opens DOWN and the x-intercepts are (4, 0) and (–2, 0).

   $f(x) = -(x-4)(x+2)$

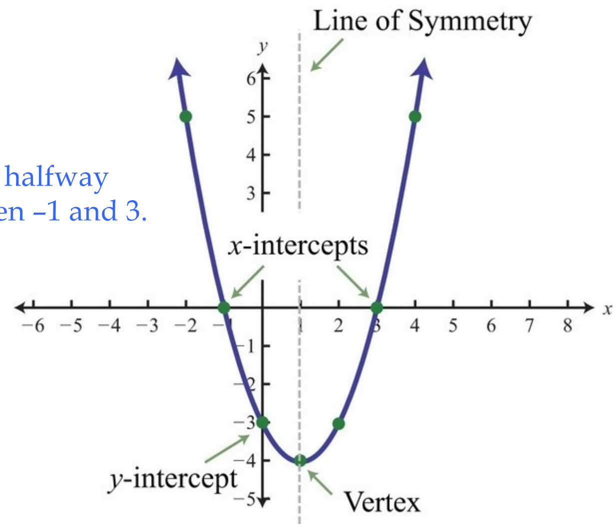
6. The parabola opens UP and the x-intercepts are (1, 0) and $\left(\frac{2}{3},0\right)$.

   $f(x)=(x-1)\left(x-\frac{2}{3}\right)$

Finding the Axis of Symmetry

The ***axis of symmetry*** is the midpoint between the *x*-coordinates of the *x*-intercepts.

1 is halfway between –1 and 3.



How do we find the axis of symmetry given the x-intercepts?

Let's Look at an Example!

Find the axis of symmetry if the x-intercepts are (–1, 0) and (3, 0)?

$$x=\frac{-1+3}{2}=\frac{2}{2}=1$$

Add the *x*-coordinates.
Divide by 2.

Thus, $x = 1$.

Determine the axis of symmetry of a parabola with the given x-intercepts.

1. The *x*-intercepts are (–12, 0) and (4, 0).

   $$x=\frac{-12+4}{2}=\frac{-8}{2}=-4$$

   Axis of Symmetry: $x = -4$

2. The *x*-intercepts are (7, 0) and (0, 0).

   $$x=\frac{7+0}{2}=\frac{7}{2}=3.5$$

   Axis of Symmetry: $x = 3.5$

3. The *x*-intercepts are (–8, 0) and (–2, 0).

   $$x=\frac{-8+(-2)}{2}=\frac{-10}{2}=-5$$

   Axis of Symmetry: $x = -5$

4. The *x*-intercepts are (-3.5, 0) and (4.1, 0).

   $$x=\frac{-3.5+4.1}{2}=\frac{0.6}{2}=0.3$$

   Axis of Symmetry: $x = 0.3$

Finding the Vertex

Follow These Steps!

1. Find the axis of symmetry (AOS). ***This is the x-coordinate of the vertex!***
2. Plug the AOS in for ***x*** and solve the quadratic equation. ***This is y-coordinate of the vertex!***

Let's Look at an Example! How do we find the vertex of a quadratic function given the x-intercepts?

Determine the vertex for a parabola given the quadratic function: $f(x) = (x + 2)(x - 2)$ and the x-intercepts $(-2, 0)$ and $(2, 0)$.

1. Find the axis of symmetry: $x = \frac{-2+2}{2} = \frac{0}{2} = 0$

2. Let $x = 0$ and solve for y (or $f(x)$): $f(0) = (0 + 2)(0 - 2) = 2 \bullet (-2) = -4$

   The vertex is $(0, -4)$.

Determine the vertex of a parabola given the quadratic function and the x-intercepts.

1. The quadratic function is $f(x) = (x + 3)(x + 1)$ and the x-intercepts are $(-3, 0)$ and $(-1, 0)$.

   $x = \frac{-3+(-1)}{2} = \frac{-4}{2} = -2$

   $f(-2) = (-2 + 3)(-2 + 1) = (1)(-1) = -1$

   The vertex is $(-2, -1)$.

2. The quadratic function is $f(x) = (x + 5)(x - 3)$ and the x-intercepts are $(-5, 0)$ and $(3, 0)$.

   $x = \frac{-5+3}{2} = \frac{-2}{2} = -1$

   $f(-1) = (-1 + 5)(-1 - 3) = (4)(-4) = -16$

   The vertex is $(-1, -16)$.

3. The quadratic function is $f(x) = (x - 2)(x - 12)$ and the x-intercepts are $(2, 0)$ and $(12, 0)$.

   $x = \frac{2+12}{2} = \frac{14}{2} = 7$

   $f(7) = (7 - 2)(7 - 12) = (5)(-5) = -25$

   The vertex is $(7, -25)$.

Graphing a Quadratic Function

Putting It All Together!

1. Use the quadratic equation written in factored form to find the x-intercepts.
2. Use the x-intercepts to find the axis of symmetry.
3. Use the axis of symmetry to find the vertex.
4. Graph all 3 points: the x-intercepts and the vertex to form a U-shaped curve called a parabola.

Quadratic Equation $\rightarrow$ x-intercepts $\rightarrow$ Axis of Symmetry $\rightarrow$ Vertex $\rightarrow$ Parabola

Let's Look at an Example! How do we graph a quadratic function?

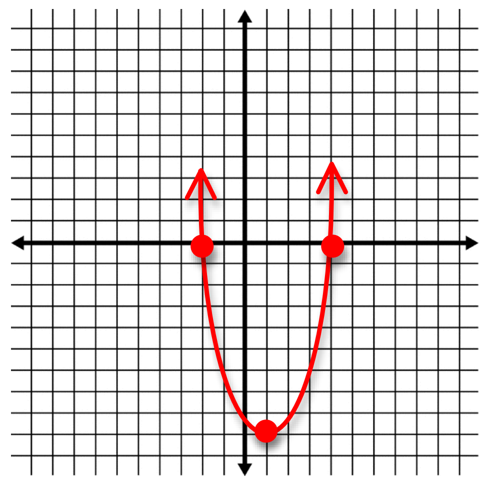
$f(x) = (x - 4)(x + 2)$

x-intercepts: (4, 0) and (–2, 0)

axis of symmetry: $x = \frac{4+(-2)}{2} = \frac{2}{2} = 1$

$f(1) = (1 - 4)(1 + 2) = (-3)(3) = -9$,

so the vertex is (1, –9)



Identify the x-intercepts and the vertex. Then, graph each of the quadratic functions.

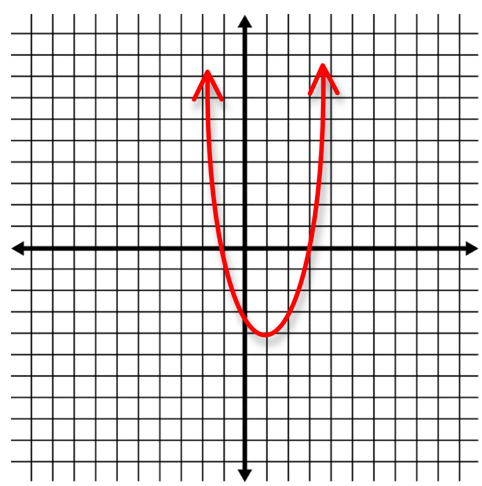
1. $f(x) = (x + 1)(x - 3)$

x-intercepts: (-1, 0) and (3, 0)

axis of symmetry: $x = \frac{-1+3}{2} = \frac{2}{2} = 1$

$f(1) = (1 + 1)(1 - 3) = (2)(-2) = -4$,

so the vertex is (1, –4)



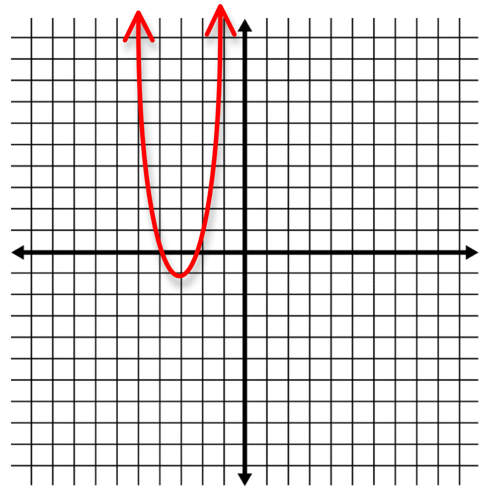
2. $f(x) = (x + 2)(x + 4)$

x-intercepts: (-2, 0) and (-4, 0)

axis of symmetry: $x = \frac{-2+(-4)}{2} = \frac{-6}{2} = -3$

$f(-3) = (-3 + 2)(-3 + 4) = (-1)(1) = -1$,

so the vertex is $(-3, -1)$



3. $f(x) = -x(x - 4)$

x-intercepts: (0, 0) and (4, 0)

axis of symmetry: $x = \frac{0+4}{2} = \frac{4}{2} = 2$

$f(2) = (-2)(2 - 4) = (-2)(-2) = 4$,

so the vertex is $(2, 4)$